Applied Stochastic Models (SS 09)
Problem Set 4

Problem 1
Find the mean and variance of the Weibull distribution with hazard rate (or failure rate)

\[ r(t) = \frac{f(t)}{1 - F(t)} = \lambda \alpha \lambda t^{\alpha - 1}, \lambda > 0, \alpha > 0, t \geq 0. \]

Problem 2
Let \( N(t) \) be a Poisson process, and let \( Y_1, Y_2, Y_3, \ldots \) be independent and identically distributed random variables. Find the mean and variance of

\[ \sum_{i=1}^{N(t)} Y_i. \]

Problem 3
Consider a homogeneous Poisson process \( N(t) \) with random (and with respect to time constant) intensity \( \lambda \) which takes two values \( \lambda_1, \lambda_2 \) with equal probabilities. (This means that first, someone rolls the intensity according to a fair coinflip, and then generates the Poisson process with this intensity.) Find the probability generating function of \( N(t) \).

Problem 4
Let the times between the events of a renewal process \( N(t) \) be uniformly distributed on \((0, 1)\). Find the mean and variance of \( N(t) \) for \( 0 < t < 1 \).