Applied Stochastic Models (SS 09)
Problem Set 11

Problem 1
Describe three distinct methods to generate a random variable with density

\[ f(x) = 6x(1-x), \quad 0 \leq x \leq 1. \]

Problem 2
Let \( X \) be a non-negative integer-valued random variable with

\[ h(r) = P(X = r|X \geq r). \]

If \( U_i, i = 1, 2, 3, \ldots \) are independent and uniform in \([0, 1]\), show that \( Z = \min\{n : U_n \leq h(n)\} \) has the same distribution as \( X \).

Problem 3
Suppose it is easy to simulate from the distributions \( F_i, i = 1, 2, \ldots, n \). Give a procedure to simulate

\[ F(x) = \sum_{i=1}^{n} p_i F_i(x), \quad p_i > 0, \sum p_i = 1. \]

Then give a method to simulate from

\[ F(x) = \begin{cases} 
\frac{1-e^{-2x}+2x}{3}, & 0 < x < 1, \\
\frac{3-e^{-2x}}{3}, & 1 < x < \infty.
\end{cases} \]

Problem 4
For a (non-homogeneous) Poisson process with intensity function \( \lambda(t), t \geq 0 \), where

\[ \int_0^\infty \lambda(t)dt = \infty, \]

let \( X_1, X_2, \ldots \) denote the sequence of times at which events occur. Show that

\[ \int_{X_{i-1}}^{X_i} \lambda(t)dt, \quad i \geq 1, \]

are independent exponential with rate 1 where \( X_0 = 0 \).