Applied Stochastic Models (SS 09)

Problem Set 13

**Problem 1**
Control charts for $\overline{X}$ and $R$ are to be set up for an important quality characteristic. The sample size is $n = 5$, and

$$\sum_{i=1}^{35} \overline{x}_i = 7805, \quad \sum_{i=1}^{35} R_i = 1200.$$  

(a) Find trial control limits for $\overline{X}$ and $R$ charts.

(b) Assuming that the process is in control, estimate the process mean and standard deviation. ($A_2 = 0.577$)

**Solution:** The grand sample mean is $\overline{x} = \frac{7805}{35} = 223$ and the mean range is $\overline{r} = \frac{1200}{35}$. Per definition of the $\overline{X}$-chart the upper control limit is

$$UCL = \overline{x} + A_2\overline{r} = 223 + 0.577 \cdot 34.286 = 242.78,$$

while the lower control limit is

$$LCL = \overline{x} - A_2\overline{r} = 223 - 0.577 \cdot 34.286 = 203.22.$$

Per definition of the $R$-chart the UCL here is given by $D_4\overline{r} = 2.115 \cdot 34.286 = 72.51$ while the LCL is given by $D_3\overline{r} = 0 \cdot 34.286 = 0$.

(b) The empirical process mean is equal to $\overline{x} = 223$ and the empirical standard deviation is just $\frac{\overline{r}}{d_2} = \frac{34.286}{2.336} = 14.74$.

**Problem 2**
A sampling plan, calling for a sample of size $n = 50$, has the acceptance number $c = 3$. Assuming that the lot size is very large, calculate the probability of accepting a lot of incoming quality 15% defective and rejecting a lot of incoming quality 4% defective

(a) by using the binomial probabilities and

(b) by using the Poisson approximation of the binomial distribution.

(c) In the above problem, use the Poisson approximation to calculate $L(p)$ for $p = 0.01, 0.02, ..., 0.20$. Sketch the OC curve for this sampling plan and read off the consumers and producers risk corresponding to an AQL of 4% and an LTPD of 14%.

**Solution:** (a) The probability of accepting a lot with incoming quality $p = 0.15$ by a sample size of 50 and acceptance number $c = 3$ is (assuming a large lot size, hence using the binomial approximation of the hypergeometric distribution) given by

$$\sum_{i=0}^{3} \binom{50}{i} p^i (1-p)^{50-i} = 0.046.$$
The probability of rejecting a lot with incoming quality \( p = 0.04 \) by a sample size of 50 and acceptance number \( c = 3 \) is given by

\[
\sum_{i=4}^{50} \binom{50}{i} p^i (1-p)^{50-i} = 0.139.
\]

(b) Using the Poisson Approximation of the binomial distribution, the above probabilities can be roughly obtained as follows: the parameter \( \lambda_1 \) is then approximately \( np_1 = 7.5 \), while \( \lambda_2 = np_2 = 2 \). The respective probabilities are

\[
\sum_{i=0}^{3} e^{-\lambda_1} \frac{\lambda_1^i}{i!} = 0.059
\]

and

\[
\sum_{i=4}^{50} e^{-\lambda_2} \frac{\lambda_2^i}{i!} = 0.142.
\]

(c) Using the Poisson approximation we find

\[
L(p) \simeq \sum_{i=0}^{3} e^{-np} \frac{(np)^i}{i!},
\]

which yields the following curve:

If the AQL is 4% then the producers risk (or type one error) is \( 1 - L(0.04) = 0.143 \) and if the LTPD is 14% then the consumers risk (or type two error) is \( L(0.14) = 0.082 \).

**Problem 3**

25 successive samples of 200 switches, each taken from a production line, contained, respectively,

6, 7, 13, 7, 0, 9, 4, 6, 0, 4, 5, 11, 6, 18, 4, 1, 9, 8, 2, 17, 9, 12, 10, 5, and 4 defectives.

If the fraction defective is to be maintained at 0.02, construct a \( p \) chart for these data and state whether or not this standard is being met.
Solution: Since there is a intended standard of \( p = 0.02 \) given, the upper and lower 3-\( \sigma \)-control limits of our \( p \)-chart are given by

\[
UCL = p + 3\sqrt{\frac{p(1-p)}{n}}
\]

and

\[
LCL = \max(p - 3\sqrt{\frac{p(1-p)}{n}}, 0).
\]

The respective sample size is \( n = 200 \), which yields \( UCL = 0.104 \) and \( LCL = 0 \). The \( p \)-chart is the following diagram and it shows that the process is in control, hence the standard is being met.

![p-chart diagram]

Problem 4
Consider the following double sampling plan. The first sample has size 15 and the acceptance number is 1, the rejection number is 5 and the second sample has size 30, the acceptance number is 5 and the rejection number is 6. If the incoming quality is \( p = 0.05 \), determine the average sample number.

Solution: The random total sample number is given by

\[
X(\omega) = 15 + 30 \mathbf{1}_A(\omega),
\]

where \( A \) is the event that a second sample is being drawn. We have

\[
P(A) = P( \text{number of defectives is 2, 3 or 4 in first sample of size 15})
\]

which is equal to

\[
\binom{15}{2} p^2(1-p)^{13} + \binom{15}{3} p^3(1-p)^{12} + \binom{15}{4} p^4(1-p)^{11} = 0.170,
\]

and hence

\[
EX = 15 + 0.170 \cdot 30 = 20.1.
\]