Abstracts

Theory of valuations and integral geometry

Semyon Alesker

Tel Aviv

In the talk we describe the notion of Radon transform on valuations. We discuss its properties in several special cases, concentrating particularly on the case of the 2-dimensional sphere.

Recent developments in tensor valuations

Andreas Bernig

Frankfurt

Tensor valuations were recently studied by McMullen, Alesker, Hug-Schneider-Schuster and others. Their main focus was on symmetric tensor valuations. In this talk, I will describe some recent progress on kinematic formulas for such valuations and on possible extensions to the non-symmetric case. I expect that non-symmetric tensor valuations will turn out useful for applications in the near future. The talk is based on joint work with Semyon Alesker and Franz Schuster on the one hand and with Daniel Hug on the other hand.

Visibility estimates in the Euclidean and hyperbolic spaces

Pierre Calka

Rouen

We consider a Poisson point process X in the Euclidean space and a family of independent copies $\{K_x : x \in X\}$ of a random convex body containing the origin. The union $\bigcup_{x\in X}(x \oplus K_x)$ is said to be the occupied phase of the classical continuum percolation model (or germ-grain model). In this talk, we are interested in the visibility function, i.e. the length of the largest segment emanating from the origin and contained in the unoccupied phase of the model. We study the distribution of this variable in any dimension. In particular, we get a two-term expansion of the logarithm of its tail probability. The method is based on precise estimates for the probability to cover the unit-sphere with a germ-grain model. We prove extreme value results in some particular asymptotic settings. We then turn to the problem of visibility percolation which consists in asking whether it is possible to see to infinity in at least one direction with positive probability. In the Euclidean case, we show a phase transition when the intensity measure of the underlying point process is modified accordingly. We conclude with extensions of this results in the hyperbolic plane and for Poisson line processes. This talk is based on joint works with J. Michel, S. Porret-Blanc and J. Tykesson.

USING SIMULATIONS TO UNDERSTAND THE ROLE OF PARTICLE SHAPE IN JAMMED GRANULAR SYSTEMS

Gary W. Delaney

Melbourne

Granular systems composed of jammed packings of non-spherical particles are found in a wide range of biological, physical and industrial systems. Through a range of different simulation methods, we investigate how the shape of the individual particles affects the physical and structural properties of the resulting granular packings. We consider systems with a range of size distributions, from mono-disperse random granular packings to Apollonian type packings where the particles have very large power-law variations in size. We demonstrate the different behaviors observed as we transition from spherical particles possessing only translational degrees of freedom to large aspect ratio non-spherical grains where rotational degrees of freedom are highly important. We quantify how broken rotational symmetry and variations in the surface curvature of the individual particles affects the tendency of the particles to order. The consequences of the results to accepted definitions for the states of static granular matter will be discussed.

COMPLEX GEOMETRY VIA A SCAFFOLD OF TRIPLY-PERIODIC MINIMAL SURFACES

Myfanwy Evans

Erlangen-Nürnberg

Networks of complex morphology can be constructed using a scaffold of Triply-Periodic Minimal Surfaces. This talk will give an overview of the EPINET database, which is a catalogue of such networks, and further detail recent extensions to include structures consisting of multiple interpenetrating networks, as well as weavings of infinite fibres.

POISSON HAIL ON A HOT GROUND

Sergey Foss

Edinburgh

We consider a queue where the server is the Euclidean space, and the customers are random closed sets (RACS) of the Euclidean space. These RACS arrive according to a Poisson rain and each of them has a random service time (in the case of hail falling on the Euclidean plane, this is the height of the hailstone, whereas the RACS is its footprint). The Euclidean space serves customers at speed 1. The service discipline is a hard exclusion rule: no two intersecting RACS can be served simultaneously and service is in the First In First Out order: only the hailstones in contact with the ground melt at speed 1, whereas the other ones are queued; a tagged RACS waits until all RACS arrived before it and intersecting it have fully melted before starting its own melting. We give the evolution equations for this queue. We prove that it is stable for a sufficiently small arrival intensity, provided the typical diameter of the RACS and the typical service time have finite exponential moments. We also discuss the percolation properties of the stationary regime of the RACS in the queue. This is a joint work with Francois Baccelli.

MORPHOLOGY AND MORPHOGENESIS OF THE DROSOPHILA EYE: INTERFACIAL MECHANICS IN BIOLOGICAL SYSTEMS

Sascha Hilgenfeldt

Illinois

Many cellular materials, both living and inanimate, are accurately described by free energy functionals dominated by interfacial terms. In previous work, we have shown that the morphology of the adult fruit fly retina can be quantitatively derived from the optimization of such a functional. Here we show how the eventual equilibrium shape depends not only on the expression levels of adhesion molecules (cadherins) in the cells, but also on the way cadherin production is regulated and on the sequence of cadherin expression and tissue remodeling events. Because of such dependences, equilibrium simulations can be used to validate or reject various hypotheses about key events in the morphogenetic development of the Drosophila eye. The regular and precise geometry and shape of cells in the Drosophila retina make this epithelial tissue an ideal starting point for this analysis, but the method is applicable to a wide variety of tissues, making it a powerful novel tool for investigating tissue formation and regeneration.

APPROXIMATION OF INTRINSIC VOLUMES AND TENSOR VALUATIONS FROM BINARY IMAGES

Markus Kiderlen

Aarhus

We discuss the problem of how geometric functionals φ (like volume, surface area, or more generally, intrinsic volumes or tensor valuations) of a set A in n-dimensional Euclidean space can be approximated, if only a binary image of the set is available. We will explain the fundamental difference of *local* and *global* approximation algorithms, the former having linear run-time and being easy to implement, whereas the latter usually yielding more accurate approximations. As binary images are finite representations of continuous sets, an error-free approximation cannot be expected. However, an approximation can be *multigrid convergent*, meaning that it converges to the correct value $\varphi(A)$ as the resolution of the digitization goes to infinity.

We will review currently used local (and some global) algorithms for the approximation of Minkowski functionals: they are are either based on discretizations of Crofton's formula, or the principal kinematic formula, or they use the polygonal approach (sometimes called "integral geometric approach") considering Minkowski functionals of a grid-based polygonal approximation of the object. Another class of methods combines morphological dilations and formation of set complements in a suitable way. Known results on multigrid convergence of these algorithms will be given.

We will then turn to tensor valuations, discussing the established polygonal approach for their approximation. Finally alternative algorithms will be suggested, e.g. using approximations of curvature or surface area measures to obtain new surface tensor algorithms.

THE STRUCTURE OF RANDOM SOAP FROTH: FROM VORONOI POLYHEDRA TO FOAM CELLS

Andrew M. Kraynik

Albuquerque (retired)

Soap froth - the quintessential foam - is composed of polyhedral gas bubbles separated by curved surfaces (thin liquid films) that satisfy Plateau's laws. Topological and geometric statistics that relate to the shape and packing of the bubbles have been explored through simulations with the Surface Evolver, a computer program developed by Brakke. The calculations are in excellent agreement with the seminal experiments by Matzke (1946) on foam structure. We will also outline the progress and problems associated with modeling the Plateau borders in random"we" foams, which have finite liquid content. Current work on the structure of random polydisperse two-dimensional foams with a wide range of liquid fraction will also be discussed.

Regularity conditions in the realisability problem and applications to point processes and random closed sets.

Raphaël Lachièze-Rey

Luxembourg

The talk addresses the existence issue for a rather general random element whose distribution is only partially specified. The technique relies on the existence of a positive extension for linear functionals accompanied by additional conditions that ensure the regularity of the extension needed for interpreting it as a probability measure. It is shown in which case the extension can be chosen to possess some invariance properties. The results are applied to obtain existence results for point processes with given correlation measure and random closed sets with given two-point covering function or contact distribution function. The regularity conditions ensure that the obtained point processes are indeed locally finite and random sets have closed realisations.

Percolation on preferential attachment networks

Peter Mörters

Bath

We study a dynamical random network model in which at every construction step a new vertex is introduced and attached to every existing vertex independently with a probability proportional to a concave function of its current degree. Such a network is called robust if it survives percolation with any positive retention parameter. We characterise the robust networks and explicitly determine the critical percolation threshold in the case of nonrobust networks. The talk is based on joint work with Steffen Dereich (Marburg).

Gaussian fluctuations on the Poisson space: from Stein's method to universality

Giovanni Peccati

Luxembourg

We shall explore some remarkable connections between the Malliavin calculus of variations on the Poisson space, and the so-called Stein's method for normal approximations. The combination of these two techniques leads to very general central limit theorems involving square-integrable functionals of Poisson measures, like for instance U-statistics. Some universality results will be also discussed. Joint works with Solé, Taqqu and Utzet (2010), Zheng (2010) and Lachièze-Rey (in preparation, 2011).

Random parking and rubber elasticity

Mathew Penrose

Bath

Joint work with Antoine Gloria (INRIA, Lille)

Renyi's random parking process on a domain D in d-space is a point process with hard-core and no-empty-space properties that are desirable for modelling materials such as rubber. It is obtained as follows: particles arrive sequentially at uniform random locations in D, and are rejected if they violate the hard-core constraint, until the accepted particles saturate D.

We describe how any real-valued functional on this point process, provided it enjoys certain subadditivity properties, satisfies an averaging property in the thermodynamic limit. Consequently in this limit, one has a convergence of macroscopically-defined energy functionals for deformations of the point process, to a homogenized limiting energy functional. We may also apply the results to derive laws of large numbers for classical optimization problems such as travelling salesman on the parking point process.

Generalized curvatures and flag measures

Jan Rataj

Praha

For general convex bodies in Euclidean spaces or, more generally, for sets with positive reach, support measures (called also generalised curvature measures or curvature-direction measures) can be introduced by using a local Steiner formula (due to Federer); their support is contained in the unit normal bundle of the body in question. Their projections to the first vector coordinate are curvature measures supported on the set boundary. For certain mixed functionals appearing in translative integral geometry, it appears that it is useful to introduce s.c. extended curvature measures living on an even larger space, namely the space of all tangent flats to the set of given dimension (or, equivalently, on the space of "flags" given by tripples (x, n, V), x being a boundary point, n a unit outer normal vector at x and V a subspace of given dimension containing n). We derive a representation of certain mixed volumes of translative integral geometry as integrals over product of extended curvature measures of the two bodies. We show also that the extended curvature measures appear naturally in the definition of absolute curvature measures for sets with positive reach (as done by Baddeley or Zähle) and in rotational integral formulas for intrinsic volumes. Joint work with Daniel Hug and Wolfgang Weil.

Stochastic models for open foams

Claudia Redenbach

Kaiserslautern

Macroscopic properties (e.g., permeability, thermal conductivity or acoustic absorption) of foams are highly influenced by the microstructure. Models from stochastic geometry are powerful tools for studying these relations. Edge systems of tessellation models are typically used to model open foams. We propose the use of random Laguerre tessellations which are weighted generalisations of the well-known Voronoi model. Using these models, a large variety of cell structures can be generated. Dilated versions of their cell edges then serve as models for the strut systems of open foams. A typical feature of real foams is a locally varying thickness of the struts. Usually, they are thicker at the vertices than at their centres. This structure is modelled using locally adaptable dilations of the edge system, where the size of the structuring element is chosen depending on the local structure. Finally, open foams are not necessarily perfectly open but may contain several closed facets. This feature can be modelled by including some of the tessellation's facets into the foam model. Since the intensity and the orientation distribution of these facets have an impact on the macroscopic properties of the foam they have to be reproduced correctly in the model. We present a model capturing the aforementioned properties of real foams. Its application is discussed by some examples. The model fitting is based on geometric characteristics of the foams which are estimated from tomographic images of the materials.

Disordered dry granular materials: between fragile solid (jamming) and dry fluid

Nick Rivier

Strasbourg

Dry granular matter, with infinite tangential friction, is modelled as a connected graph of grains linked by purely repulsive contacts. Its dynamics is associated with the adjacency matrix of the graph. The degrees of freedom of a grain are non-slip rotation on, and disconnection from another. The material stability under shear (jamming) is ensured by odd circuits of grains in contact ("arches") that prevent the grains from rolling on each other. A dense granular material with high stiffness-to-load ratio has two possible states: fragile solid, blocked by odd circuits, and dry fluid or bearing in the absence of odd circuits. The dry fluid state is pure gauge, i.e. rotation of grain A is determined uniquely from grain B, irrespective of the path connecting A and B. In the absence of odd circuits, one has a dry fluid that flows under shear by creation and glide of a pair of dislocations as in plasticity of continuous media.

We introduce the notions of *blob*, region of the material containing only even circuits, and of *critical contact* that closes an odd circuit. The core of odd vorticity constitutes closed R*-loops* that are large (size L) in *disordered* granular materials (whereas they extend over a few grains in ordered crystallisation). The disordered granular material is then represented, at low energies and critical applied shear, as a chain of blobs connected by critical contacts. The area of the minimal surface bounded by R-loops (the number of critical contacts) is the order parameter ($1/L$ per grain) of the fragile solid; the transition between dry fluid and fragile solid is a second-order phase transition with scaling laws, that occurs by intermittency (the line tension of the R-loop is negligible).

STOCHASTIC 3D MODELING OF TOMOGRAPHIC IMAGE DATA FOR GRAPHITE ELECTRODES IN LI-ION BATTERIES

Volker Schmidt

Ulm

The morphological microstructure of complex porous or composite materials is closely related to their physical properties, in particular with transport processes of gases, fluids, or charges, and with degradation mechanisms occurring in these materials. Thus, the systematic development of 'designed' morphologies with improved physical properties is an important problem which has various applications, e.g., in batteries, fuel cells, and solar cells. Mathematical models from stochastic geometry can help to solve this problem, since they can be used to provide a detailed, quantitative description of complex microstructures in existing materials. Moreover, systematic modifications of model parameters and the combination of stochastic microstructure models with numerical transport models offer the opportunity to construct new 'virtual' morphologies with improved physical properties, using model-based computer simulations.

In this talk we present a new approach to stochastic segmentation and modeling of 3D images, which show complex microstructures reconstructed by electron or synchrotron tomography. Using a multiscale approach, it is possible to decompose complex microstructures into several (less complex) components. In particular, a macroscale component is determined by morphological smoothing, which can be represented by unions of overlapping spheres. This leads to an enormous reduction of complexity and allows us to model the macroscale component by random marked point processes, which is one of the most fundamental classes of models in stochastic geometry. On the other hand, by the morphological smoothing, a small fraction of voxels is misspecified. The set of these misspecified voxels is interpreted as the microscale component of the microstructure. It is modeled separately, using random particle systems of Cox type. Finally, integrating the microscale model into the macroscale model, a complete stochastic model is obtained for geometrically complex 3D morphologies.

DISORDERED SPHERE AND ELLIPSOID PACKINGS

Gerd Schröder-Turk

Erlangen-Nürnberg

The local structure of particle ensembles is important for physical properties of normal or supercooled fluids, jammed bead packs or structural glass phases. It is often characterized by order parameters such as $q4$ or $q6$, defined by spherical harmonics of particle neighborhoods. Here we show that a Minkowski tensor analysis of the particles' Voronoi cells provides shape indices that give a clear signature of various structural transitions in particle systems. In particular, all of the above mentioned systems consist of locally anisotropic environments. We show that the degree of cell anisotropy shows a clear signature of the jamming transition in bead packs, the transition to partially ordered states at the random close packing limit, and of the transitions from fluid to ordered phases in simple liquids. The analysis is based on Minkowski tensors of rank two and rank four.

[1] Schröder-Turk et al, Europhys. Lett., 90(3), 34001 (2010)

[2] Kapfer et al, J. Stat. Mech. (2010) P11010

RECENT DEVELOPMENTS AND FUTURE PERSPECTIVES IN RANDOM TESSELLATION THEORY

Christoph Thäle

Osnabrück

Random tessellations are locally finite random subdivisions of the d-space into non-overlapping compact convex polytopes and they are one of the cornerstones in modern stochastic geometry. In my talk I discuss – from a personal point of view – some recent developments that are in particular related to i) scalar mean values and mean value relations, ii) large cells and faces and iii) the collection of the cells neighboring the typical cell. These recent developments are contrasted with several future perspectives and some selected open problems. In some more detail we discuss i) the parameter space of scalar mean values and also some mean tensors for random tessellations, ii) small cells and faces and iii) the typical neighbor of the typical cell.

LOCAL FUNCTIONALS AND BOOLEAN MODELS

Wolfgang Weil

Karlsruhe

The classical mean value formulas for Boolean models with convex or polyconvex grains concern the basic motion invariant, additive functionals, the intrinsic volumes (Minkowski functionals). The formulas have been extended to other, direction dependent, additive functionals like the mixed volumes. It seems that additivity plays a natural role here, in view of the structure of a Boolean model as the union of randomly scattered particles. Surprisingly, the additivity can be neglected. More important is the fact that the intrinsic volumes (mixed volumes) allow a local extension as measures (curvature measures, mixed curvature measures). For such local functionals, analogous mean value formulas for stationary Boolean models can be established. For polytopal grains, examples of local functionals exist which are not additive.