Exercises in Convex Geometry
Exercise Sheet No. 8 – 12/09/2008

Exercise 28
Let $K \in \mathbb{K}^n$ be a convex body, $E \subset \mathbb{R}^n$ an affine subspace such that $E \cap \text{int } K \neq \emptyset$ and let $(K_i)_{i \in \mathbb{N}}$ be a sequence in $\mathbb{K}^n$. Show that

$$K_i \to K \implies E \cap K_i \to E \cap K.$$ 

Exercise 29
a) Let $s_1, \ldots, s_n \in \mathbb{K}^n$ be segments of the form $s_i = [0, x_i], x_i \in \mathbb{R}^n$. Show that

$$n! V(s_1, \ldots, s_n) = |\det(x_1, \ldots, x_n)|.$$ 

b) Let $K_1, \ldots, K_n \in \mathbb{K}^n$ be convex bodies. Show that $V(K_1, \ldots, K_n) > 0$ if and only if there exist segments $s_i \subset K_i, i = 1, \ldots, n$, with linearly independent directions.

Exercise 30
For $K, K' \in \mathbb{K}^n$, show that

$$\int_{D(K,K')} dx = \sum_{j=0}^{n} \binom{n}{j} V(K, \ldots, K, \underbrace{-K', \ldots, -K'}_{n-j}) j,$$

where $D(K, K') := \{ z \in \mathbb{R}^n : K \cap (K' + z) \neq \emptyset \}$

Exercise 31
For a convex body $K \in \mathbb{K}^n$ and $\alpha \geq 0$, prove the following Steiner formula for the intrinsic volumes:

$$V_k(K + B(\alpha)) = \sum_{j=0}^{k} \alpha^{k-j} \binom{n-j}{n-k} \frac{\kappa_{n-j}}{\kappa_{n-k}} V_j(K) \quad (0 \leq k \leq n - 1).$$

Turn in your solutions on Tuesday, 12/16/2008, after the lecture. Do not forget to indicate your name and student identity number (Matrikelnummer) on your solutions.