Exercises in Convex Geometry
Exercise Sheet No. 11 – 01/20/2009

Exercise 39
Let $K, M, L \in \mathcal{K}^n$ be such that $K = M + L$. Show that

$$S_j(M, \cdot) = \sum_{i=0}^{j} (-1)^{j-i} \binom{j}{i} S(K, \ldots, K, L, \ldots, L, B(1), \ldots, B(1), \cdot),$$

for $j = 0, \ldots, n - 1$.

Exercise 40
Let $\alpha \in (0, 1)$ and $M, L \in \mathcal{K}^n$ with $\dim M = \dim L = n$.
Suppose $K_\alpha \in \mathcal{K}^n$ fulfills

$$S_{n-1}(K_\alpha, \cdot) = \alpha S_{n-1}(M, \cdot) + (1 - \alpha) S_{n-1}(L, \cdot).$$

Explain why $K_\alpha$ exists.
Show that

$$V(K_\alpha)^{\frac{n-1}{n}} \geq \alpha V(M)^{\frac{n-1}{n}} + (1 - \alpha) V(L)^{\frac{n-1}{n}},$$

with equality if and only if $M$ and $L$ are homothetic.

Exercise 41

a) Show that the mapping $K \mapsto \Pi K$ is continuous.

b) Show that $\Pi K$ is a polytope if $K$ is a polytope.

c) Let $K \in \mathcal{K}^2$ such that $K = -K$ and let $\vartheta$ be a rotation by $\frac{\pi}{2}$. Show that $\Pi K = \vartheta(2K)$.

Exercise 42
Let $K, L \in \mathcal{K}^n$. Show that

$$V(K, \ldots, K, \Pi L) = V(L, \ldots, L, \Pi K).$$

Turn in your solutions on Tuesday, 01/27/2009, after the lecture. Do not forget to indicate your name and student identity number (Matrikelnummer) on your solutions.