Exercises in Convex Geometry
Exercise Sheet No. 13 – 02/03/2009

Exercise 46
Let $K, K' \in \mathcal{K}^n$ and $K \cup K' \in \mathcal{K}^n$. Show that:

a) $(K \cap K') + (K' \cup K') = K + K'$.

b) $(K \cap K') + M = (K + M) \cap (K' + M)$, for all $M \in \mathcal{K}^n$.

b) $(K \cup K') + M = (K + M) \cup (K' + M)$, for all $M \in \mathcal{K}^n$.

Exercise 47
Let $\varphi(K) := V(K[j], M_{j+1}, \ldots, M_n)$, where $K, M_{j+1}, \ldots, M_n \in \mathcal{K}^n$. Show that $\varphi$ is additive, that is

$$\varphi(K \cap K') + \varphi(K \cup K') = \varphi(K) + \varphi(K')$$

for all $K, K' \in \mathcal{K}^n$ with $K \cup K' \in \mathcal{K}^n$.

Exercise 48
Show that the mappings $K \mapsto S_j(K, A)$ are additive on $\mathcal{K}^n$, for all $j \in \{0, \ldots, n\}$ and all Borel sets $A \subset S^{n-1}$.

Exercise 49
Let $\varphi : \mathcal{R}^n \to \mathbb{R}$ be additive and $A \in \mathcal{R}^n, A = \bigcup_{i=0}^{k} K_i$, $K_i \in \mathcal{K}^n$. Give a proof for the inclusion-exclusion formula

$$\varphi(A) = \sum_{v \in S(k)} (-1)^{|v|-1} \varphi(K_v).$$

Turn in your solutions on Tuesday, 02/10/2009, after the lecture. Do not forget to indicate your name and student identity number (Matrikelnummer) on your solutions.