Exercise Sheet No. 11
Advanced Mathematics II

Exercise 51:
(a) Calculate the following integrals using partial fraction decomposition

\[ \int_{0}^{\frac{1}{2}} \frac{x - 2}{x^3 + 2x^2 - x - 2} \, dx, \quad \int \frac{3x^2 + 3x + 1}{x^3 + 2x^2 + x} \, dx. \]

(b) Calculate the following integral using a substitution first and a partial fraction decomposition afterwards

\[ \int \frac{5e^{3x} + e^{2x} + 3e^x + 1}{e^{3x} + e^x} \, dx. \]

(c) Calculate the following improper integrals if they exist

\[ \int_{2}^{\infty} \frac{1}{x^5} \, dx, \quad \int_{2}^{\infty} \sin(x) + e^{-x} \, dx. \]

Exercise 52: Find the Laplace transforms of the following functions by using the identities and the table for the Laplace transform

(a) \( f(t) = 3e^{4t} + 2 \), \quad (b) \( h(t) = e^{-t} \cos(2t) \), \quad (c) \( g(t) = \begin{cases} \sin(\omega t - \varphi), & \text{if } \omega t - \varphi \geq 0, \\ 0, & \text{otherwise}. \end{cases} \)

(d) Find the Laplace transform of the following function by differentiating in the image space twice

\[ f(t) = t^2 \sin^2(t), \quad t \in [0, \infty). \]

Hint: \( \sin^2(t) = \frac{1 - \cos(2t)}{2} \).

Exercise 53:
(a) Find the Laplace transform of the following function

\[ f(t) = (3 - t^2) \sin(t) - 3t \cos(t). \]

(b) Solve the following initial value problem using a Laplace transformation

\[ u^{(4)}(t) + 2u''(t) + u(t) = \sin(t), \quad u(0) = u'(0) = u''(0) = u'''(0) = 0, \quad t \geq 0. \]

Exercise 54: Solve the following initial value problem by means of the Laplace-transform

\[ \begin{align*}
x'(t) & = 2y(t) + 1, \\
y'(t) & = -2x(t) + 2t,
\end{align*} \quad x(0) = 0, \quad y(0) = 1.\]

Exercise 55: Solve the following initial value problem using the Laplace transformation

\[ \begin{align*}
x(t) & = 2y(t) + z(t) = -2t, \\
x'(t) & + 3y'(t) - 2x(t) + y(t) = 3 + t, \\
x''(t) & - 5x'(t) - 2z(t) = 0, \\
x(0) = 1, \quad y(0) = -1, \quad z(0) = -3, \quad z'(0) = 2.
\]

Due date: Your written solutions are due on Tuesday, 8th July 2014. Please put them into the box in the student office until 2:00 PM.
Tutorial No. 11
Advanced Mathematics II

Exercise T31: Calculate the following integral by means of partial fraction decomposition

\[
\int \frac{x^3 + 6x^2 + 3x + 18}{x^4 + x^2 + 4x + 4} \, dx.
\]

Exercise T32:

(a) Find the Laplace transform of the following functions

(a1) \( f(x) = x^2 + 3x + 4 + x^2 \sin(2x) \),

(a2) \( f(x) = \begin{cases} 
\sin(x), & 0 \leq x < \pi, \\
\cos(x), & x \geq \pi, 
\end{cases} \)

(a3) \( f(x) = (e^{2x} + e^{3x}) \sin(4x) \).

(b) Let \( F(s) \) denote the Laplace transform of a function \( F(t) \). Find the inverse Laplace transform \( f \) of the following functions \( F \).

(b1) \( F(s) = \frac{2}{s^2 + 2s + 2} \),

(b2) \( F(s) = \frac{e^{-s}}{(s-2)^2} + \frac{e^{-s}}{(s+2)^2} \),

(b3) \( F(s) = \frac{1}{s}(1 - e^{-7s}) \).

Exercise T33: Find a general solution of the following system of differential equations using a Laplace transformation

\[
\begin{align*}
3y'(t) &= -y(t) + 2z(t) + \sin(t) + 3\cos(t), \\
3z'(t) &= 4y(t) + z(t) - 4\sin(t).
\end{align*}
\]

For detailed information regarding this course visit the following web page:

www.math.kit.edu/iag6/lehre/am22014s/en

Tutorial: Friday, 4th July 2014, 9:45 AM