Exercise 12:
Let \( \psi = \sum_{k \in \mathbb{Z}} a_k \chi_{[k,k+1]} \) with \( \sum_{k \in \mathbb{Z}} |k| |a_k| < \infty \) and \( \sum_{k \in \mathbb{Z}} a_k = 0 \).

(a) Show that \( \psi \) is a wavelet.
   
   Hint: Consider Lemma 2.5 with \( \beta = 1 \).

(b) Which additional restrictions on \( \{a_k\}_{k \in \mathbb{Z}} \) guarantee order 1, 2 and 3?

Exercise 13:
Let \( \psi \) be a wavelet of order \( N \in \mathbb{N} \) with compact support \([T_1, T_2]\). Define

\[
\varrho(x) = \frac{1}{(N-1)!} \int_{T_1}^{x} (x-z)^{N-1} \psi(z) \, dz.
\]

Show that \( \text{supp} \, \varrho = [T_1, T_2] \) and

\[
\frac{d^N}{dx^N} \varrho = \psi.
\]

These exercises are discussed in the problem class on Thursday, November 21, 2013.