Exercise 14:
Let \( f : \mathbb{R} \to \mathbb{R} \) be Hölder continuous of non-integer order \( \alpha > 0 \) about \( x \in \mathbb{R} \), that is, for all \( t \in \mathbb{R} \)
\[
|f(t) - p_x(t)| \leq K |x - t|^\alpha ,
\]
where \( p_x \) is a polynomial of degree \( \lfloor \alpha \rfloor \). Show that \( p_x \) is uniquely given.

Exercise 15:
Let \( \psi \in L^1(\mathbb{R}) \) be a wavelet and \( f \in L^2(\mathbb{R}) \cap L^\infty(\mathbb{R}) \). Show that for almost every \( b \in \mathbb{R} \)
\[
|W \psi f(a, b)| \leq \|W \psi f(a, \cdot)\|_{L^\infty} = O(|a|^{1/2}) \quad \text{as } a \to 0.
\]
Hint: Use Young’s inequality:
\[
\|u \ast v\|_{L^p} \leq \|u\|_{L^1}\|v\|_{L^p}, \quad u \in L^1(\mathbb{R}), \, v \in L^p(\mathbb{R}), \, 1 \leq p \leq \infty .
\]

Exercise 16:
Let \( \psi \in L^2(\mathbb{R}) \) be an even and real-valued function with
\[
0 < c_\psi = \sqrt{2\pi} \int_0^\infty \hat{\psi}(\omega) \frac{\omega}{\omega} \, d\omega < \infty .
\]
Then, for \( f \in L^1(\mathbb{R}) \cap L^2(\mathbb{R}) \) we have that
\[
f(b) = c_\psi^{-1} \int_0^\infty \langle f, T^b D^a \psi \rangle_{L^2(\mathbb{R})} a^{-3/2} \, da .
\]

These exercises are discussed in the problem class on Thursday, November 28, 2013.