The exercises will be discussed in the tutorial on Thursday, May 19, 2016.

Exercise 3.1: Continuation of Exercise 2.5
Let $\gamma > 0$ and $\eta$ be a Poisson process on $\mathbb{R}^d$ with intensity measure $\gamma \cdot \lambda^d$. Show that the estimator
$$\hat{\gamma}_B := \frac{\eta(B)}{\lambda^d(B)}, \quad B \in \mathcal{B}(\mathbb{R}^d),$$
is asymptotically normal, that is, a sequence $\left( B_n \right)_{n \in \mathbb{N}}$ of bounded Borel sets in $\mathbb{R}^d$ with $\lambda^d(B_n) \to \infty$ satisfies
$$\lim_{n \to \infty} P \left( \frac{\lambda^d(B_n)}{\gamma} \left( \frac{\eta(B_n)}{\lambda^d(B_n)} - \gamma \right) \leq t \right) = \Phi(t), \quad t \in \mathbb{R},$$
where $\Phi$ is the CDF of the standard normal distribution.

Hint: Show the convergence of the characteristic function of $\sqrt{\frac{\lambda^d(B_n)}{\gamma}} \left( \frac{\eta(B_n)}{\lambda^d(B_n)} - \gamma \right)$ to the one of the standard normal distribution.

Exercise 3.2:
Let $\mathbb{V}$ be a probability measure on $\mathbb{N}_0$ with generating function
$$G_{\mathbb{V}}(s) := \sum_{n=0}^{\infty} \mathbb{V}({n}) s^n, \quad s \in [0, 1].$$

Let $\eta$ be a mixed binomial process with mixing distribution $\mathbb{V}$ and sampling distribution $\mathbb{Q}$. Show that
$$L_\eta(u) = G_{\mathbb{V}} \left( \int e^{-u \cdot d\mathbb{Q}} \right), \quad u \in \mathbb{R}_+(\mathbb{X}).$$
Assume now that $\mathbb{V}$ is a Poisson distribution; show that the above formula is consistent with Theorem 3.8.

Exercise 3.3:
Let $\eta$ be a point process on $\mathbb{X}$. Using the convention $e^{-\infty} := 0$, the Laplace transform $L_\eta(u)$ can be defined for any $u \in \mathbb{R}_+(\mathbb{X})$. Assume now that $\eta$ is a Poisson process with intensity measure $\lambda$ and is given by $\eta = \sum_{n=1}^{\infty} \delta_{X_n}$ as in (3.3). Use Theorem 3.8 to show that
$$\mathbb{E} \left[ \prod_{n=1}^{\kappa} u(X_n) \right] = \exp \left( \int (u(x) - 1) \lambda(dx) \right),$$
for any measurable $u : \mathbb{X} \to [0, 1]$. 
Exercise 3.4:
Let $\eta$ be a proper point process and $m \in \mathbb{N}$. Prove the following equations:

(a) 
\[ \eta^{(m)}(C) = \int \ldots \int 1\{(x_1, \ldots, x_m) \in C\} \left( \eta - \sum_{j=1}^{m-1} \delta_{x_j} \right)(dx_m) \left( \eta - \sum_{j=1}^{m-2} \delta_{x_j} \right)(dx_{m-1}) \ldots \left( \eta - \delta_{x_1} \right)(dx_2) \eta(dx_1), \quad C \in \mathcal{X}^m, \]

(b) 
\[ \eta^{(m)}(B^m) = \eta(B)(\eta(B) - 1) \ldots (\eta(B) - m + 1), \quad B \in \mathcal{X}, \]

(c) 
\[ \eta^{(m)}(B_1 \times \ldots \times B_m) = \prod_{i=1}^{m} \eta(B_i), \quad B_1, \ldots, B_m \in \mathcal{X} \text{ pairwise disjoint,} \]

(d) 
\[ \eta^{(2)}(B_1 \times B_2) = \eta(B_1) \eta(B_2) - \eta(B_1 \cap B_2), \quad B_1, B_2 \in \mathcal{X}. \]