

Seminar on Number Theory

Diophantine Approximation

Seminar in Winter Semester 2026/2027

Diophantine approximation begins with a simple question: although every real number can be approximated by rational numbers, how well can this be done? The subject studies how closely numbers such as $\sqrt{2}$, π or the golden ratio can be approximated by rationals, and what this reveals about their arithmetic nature.

In this seminar, we will begin with the basic theory of rational approximation and then develop the main tools used to study it. For $x \in \mathbb{R}$, we will concentrate on finding integers p, q that satisfy

$$\left| x - \frac{p}{q} \right| < \frac{1}{q^2}$$

Topics will include continued fractions, quadratic irrationals, Pell's equation, the Gauß map, badly approximable numbers, Liouville numbers, and irrationality exponents. Throughout, particular emphasis will be given to concrete examples and calculations.

We will also explore the size and structure of sets of numbers with unusual approximation properties. This leads naturally to questions from measure theory and fractal geometry, where one studies how large or small certain exceptional sets are. In the latter parts of the series, we will also look at geometric methods involving lattices and higher-dimensional approximation.

The seminar is intended for students with a background in basic real analysis and linear algebra. Most talks will be largely self-contained, and the overall goal is to give an accessible introduction to a classical subject that lies at the intersection of number theory, geometry, dynamics, and analysis.

Preliminary meeting: 3rd of August 2026, 12:00pm, SR 2.059