Infinitely many global continua bifurcating from a single solution of an elliptic problem with a concave-convex nonlinearity

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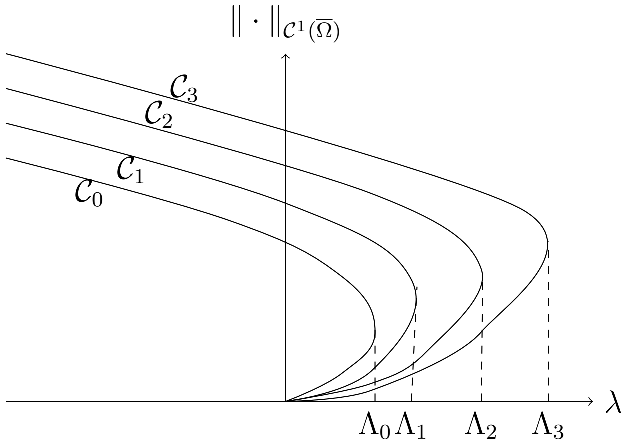
Initiated by a paper of Ambrosetti-Brézis-Cerami [1] many existence and multiplicity results for boundary value problems on bounded domains like

$$\begin{cases} -\Delta u = \lambda |u|^{q-2}u + |u|^{p-2}u & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega \end{cases} \tag{1}$$

for exponents $1 < q < 2 < p < \infty$ were found using variational methods (see [1, 3, 5]). A seminal result from [1] says that there is a positive number Λ_0 such that (1) has two positive solutions for $0 < \lambda < \Lambda_0$. Later the existence of infinitely many nontrivial solutions was proved in [3]. In a joint paper with T. Bartsch [2] we prove in the case of the annulus that these nontrivial solutions are located on mutually disjoint continua $\mathcal{C}_j$ of solutions having precisely j nodal domains which emanate from $(\lambda_0, u_0) = (0, 0)$ and satisfy $\mathrm{pr}(\mathcal{C}_j) = (-\infty, \Lambda_j]$. A study of these continua shows that they accumulate at $\mathbb{R}_{\geq 0} \times \{0\}$ and $\mathbb{R} \times \{\infty\}$ leading to bifurcation from $(\lambda, 0)$ when $\lambda \geq 0$ and bifurcation from infinity at every $\lambda \in \mathbb{R}$. The proofs are based on degree theory similar to [4]. Our method applies also to boundary value problems with more general convex-concave nonlinearities $f_\lambda(|x|, u, |\nabla u|)$.



References

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