Equivariant PDEs and the freezing method

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ABSTRACT

We consider nonlinear time dependent PDE's on unbounded domains, the solutions of which show specific spatio-temporal patterns. Examples are provided by semilinear reaction diffusion systems on $\mathbb{R}^d$, such as

$$u_t = \Delta u + f(u),\ u(x,0) = u_0(x), \quad x \in \mathbb{R}^d, t \geq 0, u(x,t) \in \mathbb{R}^m.$$

If the nonlinearity f is of excitable type such systems exhibit travelling or rotating waves for $d = 1$, rigidly rotating or meandering spiral waves for $d = 2$, and scroll waves for $d = 3$.

The idea of the *freezing method* is to determine during the numerical process a moving coordinate frame in which the aforementioned patterns become stationary. For this purpose the Cauchy problem for the PDE is transformed into a partial differential algebraic equation (PDAE). Additional algebraic variables are introduced that describe the position of the pattern, and extra constraints are imposed that try to minimize the temporal changes of the spatial profile. The method generalizes to evolution equations that are equivariant with respect to the action of a Lie group

The numerical solution of the PDAE involves several approximation processes, such as restriction to a bounded domain with asymptotic boundary conditions and discretization in space and time. We show a series of applications to systems of Ginzburg-Landau and FitzHugh-Nagumo type. Finally, we report on some analytical results related to the freezing approach. These are concerned with the preservation of asymptotic stability for specific patterns and the influence of numerical approximations on dicrete and continuous spectra of linearizations.