PERIODIC DIFFERENTIAL OPERATORS WITH ASYMPTOTICALLY PREASSIGNED SPECTRUM

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We will deal with the following operators in $\mathbb{R}^n$:

$$\mathcal{A} = -\frac{1}{b(x)} \sum_{k=1}^{n} \frac{\partial}{\partial x_k} \left(a(x) \frac{\partial}{\partial x_k} \right)$$

Here a, b are bounded above and bounded away from zero $\mathbb{Z}^n$-periodic functions. We denote by $\mathcal{L}_{\text{per}}$ the set of such operators.

In the talk we will discuss the following result obtained in [1]: for an arbitrary $L > 0$ and for arbitrary pairwise disjoint intervals $(\alpha_j, \beta_j) \subset [0, L]$, $j = 1, \ldots, m$ $(m \in \mathbb{N})$ we construct the family of operators $\{\mathcal{A}^\varepsilon \in \mathcal{L}_{\text{per}}\}_{\varepsilon>0}$ such that the spectrum of $\mathcal{A}^\varepsilon$ has exactly m gaps in $[0, L]$ when ε is small enough, and these gaps tend to the intervals (α_j, β_j) as $\varepsilon \to 0$. The corresponding functions $a^\varepsilon, b^\varepsilon$ can be chosen in such a way that their ranges have at most $m + 1$ values.

The idea how to construct the family $\{\mathcal{A}^\varepsilon\}_{\varepsilon>0}$ is based on methods of the homogenization theory.

Also we will discuss a similar result obtained for periodic Laplace-Beltrami operators [2].

References

[1] A. Khrabustovskyi, Periodic elliptic operators with asymptotically preassigned spectrum, Asymptotic Analysis (accepted). arXiv:1201.3729.

[2] A. Khrabustovskyi, Periodic Riemannian manifold with preassigned gaps in spectrum of Laplace-Beltrami operator, Journal of Differential Equations 252 (3) (2012), 2339-2369.

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