DISCRETE SPECTRUM OF SCHRÖDINGER OPERATORS WITH δ-INTERACTIONS ON CONICAL SURFACES IS INFINITE

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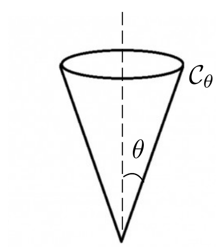
Spectral analysis of multi-dimensional Schrödinger operators with interactions supported on sets of null Lebesgue measure such as points, curves and surfaces, is a classical topic of mathematical physics. Besides physical motivation to investigate these operators stemming, in particular, from quantum mechanics, there is also a purely mathematical motivation, since these operators exhibit non-trivial interplay between their spectral properties and geometry of the interaction support.

One such an interplay will be considered in my talk based on the joint work [BEL] with Jussi Behrndt (Graz) and Pavel Exner (Prague). I will rigorously define the self-adjoint lower-semibounded three-dimensional Schrödinger operator $H_{\alpha,\theta}$ with attractive δ-interaction of constant strength $\alpha > 0$ supported on the conical surface

$$\mathcal{C}_\theta := \big\{(x,y,z) \in \mathbb{R}^3 \colon z = \cot(\theta)\sqrt{x^2+y^2}\big\}, \quad \text{with } \theta \in (0, \pi/2].$$



It turns out that the essential spectrum of this operator is independent of θ and always coincides with the set $[-\alpha^2/4, +\infty)$, whereas the discrete spectrum below the threshold is either empty if the angle $\theta = \pi/2$ or infinite if $\theta \in (0, \pi/2)$. Asymptotic estimates for the eigenvalues of $H_{\alpha,\theta}$ with $\theta \in (0, \pi/2)$ are also obtained. Our proofs rely on Dirichlet-Neumann bracketing and on variational principles for self-adjoint operators. The test functions for the variational principles should be chosen to respect the shape of $\mathcal{C}_\theta$.

Our results remain valid if the hypersurface $\mathcal{C}_\theta$ is locally deformed in an arbitrary way with some of its regularity preserved. This, in particular, shows that the tip of the cone is not "responsible" for the infiniteness of the discrete spectrum and that the corresponding effect is generated only by the shape of the conical surface $\mathcal{C}_\theta$ at infinity.

References

[BEL] J. Behrndt, P. Exner, and V. Lotoreichik, Schrödinger operators with δ-interactions supported on conical surfaces, *J. Phys. A: Math. Theor.* **47** (2014), 355202 (16pp).