Non-stationary abstract Friedrichs systems

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Abstract. Symmetric positive systems (Friedrichs systems) of first-order linear partial differential equations were introduced by Kurt Otto Friedrichs (1958) in an attempt to treat the equations that change their type, like the equations modelling transonic fluid flow. A Friedrichs system consists of a first order system of partial differential equations (of a specific type) and an *admissible* boundary condition. Friedrichs showed that this class of problems encompasses a wide variety of classical and neoclassical initial and boundary value problems for various linear partial differential equations.

More recently, Ern, Guermond and Caplain (2007) suggested another approach to the Friedrichs theory, which was inspired by their interest in the numerical treatment of Friedrichs systems. They expressed the theory in terms of operators acting in abstract Hilbert spaces and proved well-posedness result in this abstract setting. Although some evolution (non-stationary) problems can be treated within this framework, their theory is not suitable for problems like the initial-boundary value problem for the non-stationary Maxwell system, or the Cauchy problem for the symmetric hyperbolic system.

This motivates the interest in non-stationary Friedrichs systems. Some numerical treatment of such problems was already done by Burman, Ern and Fernandez (2010), although the existence and uniqueness result was missing.

We prove well-posedness result for an abstract non-stationary Friedrichs systems. To be precise, we consider an abstract Cauchy problem in a Hilbert space, that involves a time independent abstract Friedrichs operator. We use the semigroup theory approach, and prove that the operator involved satisfies the conditions of the Hille-Yosida generation theorem. We also address the semilinear problem and apply the new results to symmetric hyperbolic systems, the unsteady Maxwell system, the unsteady div-grad problem, and the wave equation. We also consider an extension of this theory to the complex Banach space setting, with application to the Dirac system.

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