Numerical Homogenization for Wave Propagation in Highly Oscillatory Media

Christian Stohrer

Abstract

The maximal admissible meshsize to approximate wave propagation through a heterogeneous medium with classical numerical schemes, such as finite differences and finite elements, is restricted by two factors: the wavelength λ and the size of the smallest heterogeneity of the medium, denoted by ε. We are particularly interested in the case where $\varepsilon \ll \lambda$. Thus, the application of classical methods requires the use of unfeasibly small mesh sizes. On the other hand, for the representation of the macroscopic behavior of the solution, i.e., on a length scale much bigger than ε, a coarser mesh would be enough.

In this talk we present the Finite Element Heterogeneous Multiscale Method (FE-HMM) and apply it to the Helmholtz and the acoustic wave equation. The FE-HMM approximates the homogenization limit, which describes well the overall behavior of the true multiscale solution of these two equations, allows the use of mesh sizes much larger than ε, and does not require a-priori knowledge of the homogenized coefficients. We prove convergence estimates and show numerical experiments corroborating our results.

This is joint work with Assyr Abdulle, Marcus J. Grote (acoustic wave equation), and Patrick Ciarlet (Helmholtz equation).

References

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