Aufgabe 3

$$x(u^1,u^2) = \left((u^2)^2 \cos u^1;\ (u^2)^2 \sin u^1;\ u^2\right)$$

$$x_{u^1} = \left(-(u^2)^2 \sin u^1;\ (u^2)^2 \cos u^1;\ 0\right) \qquad g_{11} = (u^2)^4\ ,\ g_{12} = 0$$

$$x_{u^2} = \left(+2u^2 \cos u^1,\ 2u^2 \sin u^1,\ 1\right) \qquad g_{22} = 4(u^2)^2 + 1$$

$$x_{u^1} \times x_{u^2} = \left((u^2)^2 \cos u^1,\ (u^2)^2 \sin u^1;\ -2(u^2)^3\right)$$

$$= (u^2)^2 \left(\cos u^1,\ \sin u^1;\ -2u^2\right)$$

$$n = \frac{1}{\sqrt{1+4(u^2)^2}} \left(\cos u^1; \sin u^1; -2u^2\right)$$

$$x_{u^1u^1} = \left(-(u^2)^2 \cos u^1,\ -(u^2)^2 \sin u^1;\ 0\right) \qquad b_{11} = \frac{-(u^2)^2}{\sqrt{1+4(u^2)^2}}$$

$$x_{u^1u^2} = \left(-2u^2 \sin u^1;\ 2u^2 \cos u^1,\ 0\right) \qquad b_{12} = 0$$

$$x_{u^2u^2} = \left(2\cos u^1,\ 2\sin u^1;\ 0\right) \qquad b_{22} = \frac{2}{\sqrt{1+4(u^2)^2}}$$

$$K = \frac{b}{g} = \frac{b_{11}b_{22}}{g_{11}g_{22}} = \frac{-2(u^2)^2}{(1+4(u^2)^2)} \cdot \frac{1}{(u^2)^4\,(4(u^2)^2+1)}$$

$$= \frac{-2}{(u^2)^2(1+4(u^2)^2)^2} < 0 \quad \forall (u^1,u^2) \in U$$

$$H = \frac{1}{2g}\left(g_{11}b_{22} - 2\underbrace{g_{12}b_{12}}_{=0} + g_{22}b_{11}\right) =$$

$$= \frac{1}{2(u^2)^4(1+4(u^2)^2)} \left(\frac{2(u^2)^4}{\sqrt{1+4(u^2)^2}} + \frac{(1+4(u^2)^2)(-(u^2)^2)}{\sqrt{1+4(u^2)^2}} \right)$$

$$= \frac{-(u^2)^2\ (1+4(u^2)^2 - 2(u^2)^2)}{2(u^2)^4\ (1+4(u^2)^2)^{3/2}} = \frac{-(1+2(u^2)^2)}{2(u^2)^2\,(1+4(u^2)^2)^{3/2}}$$

$$H^2 - K = \frac{(1+2(u^2)^2)^2}{4(u^2)^4(1+4(u^2)^2)^3} + \frac{2\cdot 4(u^2)^2(1+4(u^2)^2)}{4(u^2)^4(1+4(u^2)^2)^3}$$

$$\kappa_{1,2} = H \pm \sqrt{H^2 - K}$$

$$= \frac{-(1+2(u^2)^2)}{2(u^2)^2(1+4(u^2)^2)^{3/2}} \pm \frac{\sqrt{1+12(u^2)^2+36\cdot(u^2)^4}}{2(u^2)^2(1+4(u^2)^2)^{3/2}}$$

$$= \frac{-(1+2(u^2)^2) \pm (1+6(u^2)^2)}{2(u^2)^2(1+4(u^2)^2)^{3/2}}$$

$$= \frac{-1 \pm 1 - 2(u^2)^2 \pm 6(u^2)^2}{2(u^2)^2(1+4(u^2)^2)^{3/2}} \Longrightarrow \kappa_1 = \frac{2}{(1+4(u^2)^2)^{3/2}}, \quad \kappa_2 = \frac{-1}{(u^2)^2\sqrt{1+4(u^2)^2}}$$

Nach den Rechnungen zu Aufgabe 3 auf Blatt 8 wird die ONB gegeben durch $\left(0, \frac{1}{\sqrt{g_{22}}}\right)$ und $\left(\frac{1}{\sqrt{g_{11}}}, 0\right)$ (im Falle $g_{12} = g_{21} = 0$ und $b_{12} = b_{21} = 0$), also

$$x_{\xi_{(1)}} = \frac{1}{\sqrt{g_{22}}} x_{u^2} = \frac{1}{\sqrt{4(u^2)^2+1}} x_{u^2}$$

$$x_{\xi_{(2)}} = \frac{1}{\sqrt{g_{11}}} x_{u^1} = \frac{1}{(u^2)^2} x_{u^1}$$

Dort hatten wir auch eine Formel für κ_1 und κ_2.

$$\kappa_1 = \kappa_n(x_{\xi_{(1)}}) = \frac{b_{22}}{g_{22}} = \frac{2}{(1+4(u^2)^2)^{3/2}}$$

$$\kappa_2 = \kappa_n(x_{\xi_{(2)}}) = \frac{b_{11}}{g_{11}} = \frac{-1}{(u^2)^2\sqrt{1+4(u^2)^2}}.$$