7.102 Table 1. (Compact) irreducible symmetric spaces of type I

$\mathfrak{g}$	$\mathfrak{k}$	dim M	Condition	$\mathfrak{g}$	$\mathfrak{k}$	dim M
$\mathfrak{su}(p+q)$	$\mathfrak{su}(p) \oplus \mathfrak{su}(q) \oplus \mathbb{R}$	$2pq$	$1 \leqslant p \leqslant q$	E_6	$\mathfrak{su}(6) \oplus \mathfrak{su}(2)$	40
$\mathfrak{su}(n)$	$\mathfrak{so}(n)$	$\frac{(n-1)(n+2)}{2}$	$3 \leqslant n$	E_6	$\mathfrak{so}(10) \oplus \mathbb{R}$	32
$\mathfrak{su}(2n)$	$\mathfrak{sp}(n)$	$(n-1)(2n+1)$	$2 \leqslant n$	E_6	$\mathfrak{sp}(4)$	42
$\mathfrak{so}(2n)$	$\mathfrak{su}(n) \oplus \mathbb{R}$	$n(n-1)$	$5 \leqslant n$	E_6	F_4	26
$\mathfrak{so}(p+q)$	$\mathfrak{so}(p) \oplus \mathfrak{so}(q)$	pq	$1 \leqslant p \leqslant q$ $7 \leqslant p+q$	E_7	$\mathfrak{su}(8)$	70
$\mathfrak{sp}(n)$	$\mathfrak{su}(n) \oplus \mathbb{R}$	$n(n+1)$	$2 \leqslant n$	E_7	$\mathfrak{so}(12) \oplus \mathfrak{su}(2)$	64
$\mathfrak{sp}(p+q)$	$\mathfrak{sp}(p) \oplus \mathfrak{sp}(q)$	$4pq$	$1 \leqslant p \leqslant q$	E_7	$E_6 \oplus \mathbb{R}$	54
G_2	$\mathfrak{su}(2) \oplus \mathfrak{su}(2)$	8		E_8	$\mathfrak{so}(16)$	128
F_4	$\mathfrak{so}(9)$	16		E_8	$E_7 \oplus \mathfrak{su}(2)$	112
F_4	$\mathfrak{sp}(3) \oplus \mathfrak{su}(2)$	28				

7.103 Table 2. (Compact) irreducible symmetric spaces of type II

$\mathfrak{k}$	dim M	Condition	$\mathfrak{k}$	dim M
$\mathfrak{su}(n)$	n^2-1	$2 \leqslant n$	F_4	52
$\mathfrak{so}(n)$	$\frac{n(n-1)}{2}$	$7 \leqslant n$	E_6	78
$\mathfrak{sp}(n)$	$n(2n+1)$	$2 \leqslant n$	E_7	133
G_2	14		E_8	248

Notes

(i) Here $\mathfrak{g} = \mathfrak{k} \oplus \mathfrak{k}$, and the embedding $\mathfrak{k} \subset \mathfrak{g}$ is the diagonal one.

(ii) M is a (connected) simple compact Lie group.

7.104 Table 3. (Non-compact) irreducible symmetric spaces of type III

$\mathfrak{g}$	$\mathfrak{k}$	dim M	Condition	$\mathfrak{g}$	$\mathfrak{k}$	dim M
$\mathfrak{su}(p,q)$	$\mathfrak{su}(p) \oplus \mathfrak{su}(q) \oplus \mathbb{R}$	$2pq$	$1 \leqslant p \leqslant q$	E_6^2	$\mathfrak{su}(6) \oplus \mathfrak{su}(2)$	40
$\mathfrak{sl}(n, \mathbb{R})$	$\mathfrak{so}(n)$	$\frac{(n-1)(n+2)}{2}$	$3 \leqslant n$	E_6^{-14}	$\mathfrak{so}(10) \oplus \mathbb{R}$	32
$\mathfrak{sl}(n, \mathbb{H})$	$\mathfrak{sp}(n)$	$(n-1)(2n+1)$	$2 \leqslant n$	E_6^6	$\mathfrak{sp}(4)$	42
$\mathfrak{so}(n, \mathbb{H})$	$\mathfrak{su}(n) \oplus \mathbb{R}$	$n(n-1)$	$5 \leqslant n$	E_6^{-26}	F_4	26
$\mathfrak{so}(p,q)$	$\mathfrak{so}(p) \oplus \mathfrak{so}(q)$	pq	$1 \leqslant p \leqslant q$ $7 \leqslant p+q$	E_7^7	$\mathfrak{su}(8)$	70
$\mathfrak{sp}(n, \mathbb{R})$	$\mathfrak{su}(n) \oplus \mathbb{R}$	$n(n+1)$	$2 \leqslant n$	E_7^{-5}	$\mathfrak{so}(12) \oplus \mathfrak{su}(2)$	64
$\mathfrak{sp}(p,q)$	$\mathfrak{sp}(p) \oplus \mathfrak{sp}(q)$	$4pq$	$1 \leqslant p \leqslant q$	E_7^{-25}	$E_6 \oplus \mathbb{R}$	54
G_2^2	$\mathfrak{su}(2) \oplus \mathfrak{su}(2)$	8		E_8^8	$\mathfrak{so}(16)$	128
F_4^{-20}	$\mathfrak{so}(9)$	16		E_8^{-24}	$E_7 \oplus \mathfrak{su}(2)$	112
F_4^4	$\mathfrak{sp}(3) \oplus \mathfrak{su}(2)$	28				

Notes
(i) Here $M = G/K$ where G is a non-compact absolutely simple real Lie group and K is a maximal compact subgroup.
(ii) The duality defined in 7.83 interchanges spaces of type I and III in the same order.

7.105 Table 4. (Non-compact) irreducible symmetric spaces of type IV

$\mathfrak{g}$	$\mathfrak{k}$	dim M	Condition	$\mathfrak{g}$	$\mathfrak{k}$	dim M
$\mathfrak{sl}(n, \mathbb{C})$	$\mathfrak{su}(n)$	$n^2 - 1$	$2 \leqslant n$	$F_4^{\mathbb{C}}$	F_4	52
$\mathfrak{so}(n, \mathbb{C})$	$\mathfrak{so}(n)$	$\frac{n(n-1)}{2}$	$7 \leqslant n$	$E_6^{\mathbb{C}}$	E_6	78
$\mathfrak{sp}(n, \mathbb{C})$	$\mathfrak{sp}(n)$	$n(2n+1)$	$2 \leqslant n$	$E_7^{\mathbb{C}}$	E_7	133
$G_2^{\mathbb{C}}$	G_2	14		$E_8^{\mathbb{C}}$	E_8	248

Notes
(i) Here $M = G/K$ where G is a simple complex Lie group and K a maximal compact subgroup.
(ii) The duality defined in 7.83 interchanges spaces of type II and IV in the same order.