Karlsruhe Institute of Technology

Lie groups and Lie algebras
Winter semester 18/19

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1. Exercise sheet

This sheet will be discussed on 26.10.2018

Exercise 1

Show that the Lie bracket of two left-invariant vector fields is again a left-invariant vector field, i.e. for left-invariant vector fields X, Y, it holds

$$\mathrm{d}L_g X = X \circ L_g \quad \text{and} \quad \mathrm{d}L_g Y = Y \circ L_g$$

$$\Rightarrow \quad \mathrm{d}L_g [X, Y] = [X, Y] \circ L_g.$$

Exercise 2

Show that the following examples are Lie groups with trivial Lie algebra for $n, m \in \mathbb{N}$. This means all Lie brackets are 0.

- $(\mathbb{R}^n, +)$
- $T^n := \mathbb{R}^n / \mathbb{Z}^n$
- $\mathbb{R}^n \times T^m$

Exercise 3

a) Classify all 1-dimensional Lie algebras.

b) Classify all 2-dimensional Lie algebras.