

## 10. Exercise sheet

This sheet will be discussed on 11.01.2019

### Exercise 1

Let  $\mathfrak{g}$  be a real Lie algebra with trivial center.

Show that there is a linear Lie group  $G$  with Lie algebra isomorphic to  $\mathfrak{g}$ .

### Exercise 2

a) Show that  $T^n = \text{diag}(R(\theta_1), \dots, R(\theta_n))$  with

$$R(\theta) = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$

is a maximal torus in  $\text{SO}(2n)$ .

b) Show that  $\text{diag}(\pm 1, \dots, \pm 1) \cap \text{SO}(n)$  is maximal abelian in  $\text{SO}(n)$  but not contained in a maximal torus.

### Exercise 3

a) Complete the details of the proof of the corollary to theorem 6:

A compact Lie group with finite center is semisimple.

b) Give an example of such a group.