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2. Exercise sheet

This sheet will be discussed on 02.11.2018

Exercise 1

Give a complete description of the tangent bundle, the vector fields and the left invariant vector fields on the 1-dimensional Lie groups $(\mathbb{R}, +)$ and $\mathrm{SO}(2)$.

Exercise 2

Show that the exponential map for $\mathrm{SO}(2)$ is surjective but not injective.

Exercise 3

Let e be the matrix exponential map defined by

$$e : \mathfrak{gl}(n, \mathbb{R}) \to \mathrm{GL}(n, \mathbb{R}), \quad A \mapsto e^A = \sum_{k=0}^{\infty} \frac{1}{k!} A^k.$$

Prove the following claims for $A \in \mathfrak{gl}(n, \mathbb{R})$ and $B \in \mathrm{GL}(n, \mathbb{R})$:

a) $e^{BAB^{-1}} = Be^A B^{-1}$.

b) If $\lambda_1, \ldots, \lambda_n$ are the eigenvalues of A, then $e^{\lambda_1}, \ldots, e^{\lambda_n}$ are the eigenvalues of e^A.

c) $\det(e^A) = e^{\mathrm{trace}(A)}$.