

4. Exercise sheet

This sheet will be discussed on 16.11.2018

Exercise 1

Let G be a Lie group and H be a Lie subgroup of G where both are connected.
Then H is normal in G (i.e. $gHg^{-1} \subset H$ for all $g \in G$) if and only if \mathfrak{h} is an ideal in \mathfrak{g} (i.e. $[\mathfrak{g}, \mathfrak{h}] \subset \mathfrak{h}$).

Exercise 2

Show that

$$\det : \mathrm{GL}(n, \mathbb{R}) \rightarrow (\mathbb{R} \setminus \{0\}, \cdot)$$

is a Lie group homomorphism with

$$d(\det) = \text{trace}.$$

Exercise 3

Classify all 2-dimensional Lie groups.