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5. Exercise sheet

This sheet will be discussed on 23.11.2018

Exercise 1

a) Show that

$$\exp : \mathfrak{gl}(n,\mathbb{C}) \to \mathrm{GL}(n,\mathbb{C})$$

is onto, but that

$$\exp : \mathfrak{gl}(n,\mathbb{R}) \to \mathrm{GL}(n,\mathbb{R})^+ = \left\{ A \in \mathrm{GL}(n,\mathbb{R}) \;\middle|\; \det(A) > 0 \right\}$$

is not.

b) Determine the image of

$$\exp : \mathfrak{sl}(2,\mathbb{R}) \to \mathrm{SL}(2,\mathbb{R}).$$

Exercise 2

Define for

$$I_{2,1} := \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \in \mathbb{R}^{3\times 3}$$

the set

$$\mathrm{SO}(2,1) := \left\{ A \in \mathrm{SL}(3,\mathbb{R}) \;\middle|\; A I_{2,1} A^T = I_{2,1} \right\}.$$

a) Show that $\mathrm{SO}(2,1)$ is a 3-dimensional Lie group and determine its Lie algebra.

b) Show that $\mathrm{SO}(2,1)$ is locally isomorphic to $\mathrm{SL}(2,\mathbb{R})$.

Exercise 3

Show that the center $Z(G)$ of a Lie group G is a Lie subgroup with Lie algebra

$$\mathfrak{z}(\mathfrak{g}) = \left\{ X \in \mathfrak{g} \middle| ad_X = 0 \right\}.$$