Karlsruhe Institute of Technology

Lie groups and Lie algebras
Winter semester 18/19

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6. Exercise sheet

This sheet will be discussed on 30.11.2018

Exercise 1

Determine the Lie algebra and the dimension of the symplectic group $\mathrm{Sp}(n, \mathbb{C})$.

Exercise 2

Show that the Killing form of $\mathfrak{gl}(n, \mathbb{R})$ or $\mathfrak{gl}(n, \mathbb{C})$ is given by

$$B(X, X) = 2n\mathrm{tr}\left(X^2\right) - 2(\mathrm{tr}(X))^2.$$

Exercise 3

a) Let $\mathfrak{h} \subseteq \mathfrak{g}$ be an ideal (i.e. $[\mathfrak{h}, \mathfrak{g}] \subseteq \mathfrak{h}$)). Show that the Killing form of $\mathfrak{h}$ is the restriction of the Killing form of $\mathfrak{g}$.

b) Show that the Killing form of $\mathfrak{sl}(n, \mathbb{R})$ or $\mathfrak{sl}(n, \mathbb{C})$ is given by

$$B(X, X) = 2n\mathrm{tr}\left(X^2\right).$$