Karlsruhe Institute of Technology

Lie groups and Lie algebras
Winter semester 18/19

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7. Exercise sheet

This sheet will be discussed on 7.12.2018

Exercise 1

Show that in the polar decomposition of $A \in \mathrm{O}(p,q)$ (with $p, q \geq 1$) one has $R \in \mathrm{O}(p) \times \mathrm{O}(q)$. Conclude that these groups are non-compact and determine their number of connected components.

Exercise 2

Let (V, ω) be a finite dimensional symplectic vector space.
Show that there exists a symplectic basis, i.e. a basis $\{v_1, \ldots, v_n, w_1, \ldots, w_n\}$ of V such that for all $i, j = 1, \ldots, n$

$$\omega(v_i, v_j) = 0 = \omega(w_i, w_j) \qquad \text{and} \qquad \omega(v_i, w_j) = \delta_{ij}.$$

Exercise 3

Show that $\mathrm{Sp}(1, \mathbb{R})$ is locally isomorphic to $\mathrm{SL}(2, \mathbb{R})$ and $\mathrm{Sp}(1, \mathbb{C})$ to $\mathrm{SL}(2, \mathbb{C})$.