9. Exercise sheet
This sheet will be discussed on 21.12.2018

Exercise 1
Show that the Lie algebras $\mathfrak{sl}(2, \mathbb{R}), \mathfrak{sl}(2, \mathbb{C}), \mathfrak{so}(3)$ and $\mathfrak{su}(3)$ are simple.

Exercise 2
The isometry group of the euclidean plane $\mathbb{E}^2$ consists of rotations and translations and can be realized in matrix form as

$$\text{Iso}(\mathbb{E}^2) = \left\{ \begin{pmatrix} A & x \\ 0 & 0 \end{pmatrix} \in \text{GL}(3, \mathbb{R}) \mid A \in O(2), x, y \in \mathbb{R} \right\}.$$  

Determine the connected component of the identity $\text{Iso}^0(\mathbb{E}^2)$ of $\text{Iso}(\mathbb{E}^2)$ and show that it is a solvable Lie group.

Exercise 3
Give an example of a Lie algebra which is not nilpotent but whose Killing form vanishes.