

## 9. Exercise sheet

This sheet will be discussed on 21.12.2018

### Exercise 1

Show that the Lie algebras  $\mathfrak{sl}(2, \mathbb{R})$ ,  $\mathfrak{sl}(2, \mathbb{C})$ ,  $\mathfrak{so}(3)$  and  $\mathfrak{su}(3)$  are simple.

### Exercise 2

The isometry group of the euclidean plane  $\mathbb{E}^2$  consists of rotations and translations and can be realized in matrix form as

$$\text{Iso}(\mathbb{E}^2) = \left\{ \begin{pmatrix} A & x \\ 0 & 0 & 1 \end{pmatrix} \in \text{GL}(3, \mathbb{R}) \mid A \in O(2), x, y \in \mathbb{R} \right\}.$$

Determine the connected component of the identity  $\text{Iso}^0(\mathbb{E}^2)$  of  $\text{Iso}(\mathbb{E}^2)$  and show that it is a solvable Lie group.

### Exercise 3

Give an example of a Lie algebra which is not nilpotent but whose Killing form vanishes.