Karlsruher Institut für Technologie

Institute for Algebra and Geometry
HDoz.Dr. Oliver Baues
Dipl.Math. Johannes Riesterer

Riemannian Geometry

Summer Term 2011

Exercise Sheet 05

11.05.2011

Exercise 1

Let $\pi : E \to N$ be a vector bundle and $f : M \to N$ a smooth map.

a) Show that there exists a vector bundle $\pi' : E' \to M$ which satisfies the property:

  ($*$) There exists a bundle map $F : E' \to E$ over f such that $F_x : E'_x \to E_{f(x)}$ is an isomorphism for all $x \in M$.

b) Show that E' is uniquely determined up to isomorphism of vector bundles by property ($*$).

Hint: Consider $E' := \{(x, e) \in M \times E \mid f(x) = \pi(e)\}$.

Exercise 2

Let $M := [0, 1] \times \mathbb{R}/\sim$ be the Möbius strip as defined in Exercise 2 on Exercise Sheet 01. Show that M is the total space of a non trivial line bundle over the circle S^1.

Exercise 3

Let N be a smooth manifold and $M \subset N$ be a closed submanifold. If $g \in \mathcal{C}^\infty(M)$ then there exists $f \in \mathcal{C}^\infty(N)$ such that $f_{|M} = g$.

Hint: Use a partition of unity to construct f from local data.
