Karlsruher Institut für Technologie

Institute for Algebra and Geometry
HDoz.Dr. Oliver Baues
Dipl.Math. Johannes Riesterer

Riemannian Geometry — Summer Term 2011

Exercise Sheet 10 — **15.06.2011**

Exercise 1

Let M and M' be two Riemannian manifolds with Levi-Civita connections ∇ and ∇' and $\phi : M \to M'$ a diffeomorphism. Define the pullback

$$(\phi^*\nabla')_X Y := \phi^* \nabla'_{\phi_* X} \phi_* Y$$

for all $X, Y \in \mathrm{Vect}(M)$.

a) Show that $(\phi^*\nabla')$ is a connection on M.

b) Show that if ϕ is an isometry then $(\phi^*\nabla') = \nabla$.

b) Show that an isometry maps geodesics to geodesics.

c) Let $\mathcal{D}$ be the canonical connection on $\mathbb{R}^n$ and $A : \mathbb{R}^n \to \mathbb{R}^n$ a diffeomorphism. Show that $(A^*\mathcal{D}) = \mathcal{D}$ holds if and only if A is an affine map.

Exercise 2

a) Let $\mathbb{H}^2$ be the upper half plane as defined in Exercise 2 on Exercise sheet 09. Show that the $y-$axis is the image of a geodesic in $\mathbb{H}^2$.

b) Show that the half lines orthogonal to the $x-$axis and the half circles with center on the $y-$axis are precisely the images of geodesics in $\mathbb{H}^2$.
