

7.104 Table 3. (Non-compact) irreducible symmetric spaces of type III

$\mathfrak{g}$	$\mathfrak{k}$	$\dim M$	Condition	$\mathfrak{g}$	$\mathfrak{k}$	$\dim M$
$\mathfrak{su}(p, q)$	$\mathfrak{su}(p) \oplus \mathfrak{su}(q) \oplus \mathbb{R}$	$2pq$	$1 \leq p \leq q$	$E_6^2$	$\mathfrak{su}(6) \oplus \mathfrak{su}(2)$	40
$\mathfrak{sl}(n, \mathbb{R})$	$\mathfrak{so}(n)$	$\frac{(n-1)(n+2)}{2}$	$3 \leq n$	$E_6^{-14}$	$\mathfrak{so}(10) \oplus \mathbb{R}$	32
$\mathfrak{sl}(n, \mathbb{H})$	$\mathfrak{sp}(n)$	$(n-1)(2n+1)$	$2 \leq n$	$E_6^6$	$\mathfrak{sp}(4)$	42
$\mathfrak{so}(n, \mathbb{H})$	$\mathfrak{su}(n) \oplus \mathbb{R}$	$n(n-1)$	$5 \leq n$	$E_6^{-26}$	$F_4$	26
$\mathfrak{so}(p, q)$	$\mathfrak{so}(p) \oplus \mathfrak{so}(q)$	$pq$	$1 \leq p \leq q$ $7 \leq p+q$	$E_7^7$	$\mathfrak{su}(8)$	70
$\mathfrak{sp}(n, \mathbb{R})$	$\mathfrak{su}(n) \oplus \mathbb{R}$	$n(n+1)$	$2 \leq n$	$E_7^{-5}$	$\mathfrak{so}(12) \oplus \mathfrak{su}(2)$	64
$\mathfrak{sp}(p, q)$	$\mathfrak{sp}(p) \oplus \mathfrak{sp}(q)$	$4pq$	$1 \leq p \leq q$	$E_7^{-25}$	$E_6 \oplus \mathbb{R}$	54
$G_2^2$	$\mathfrak{su}(2) \oplus \mathfrak{su}(2)$	8		$E_8^8$	$\mathfrak{so}(16)$	128
$F_4^{-20}$	$\mathfrak{so}(9)$	16		$E_8^{-24}$	$E_7 \oplus \mathfrak{su}(2)$	112
$F_4^4$	$\mathfrak{sp}(3) \oplus \mathfrak{su}(2)$	28				

Notes

- (i) Here  $M = G/K$  where  $G$  is a non-compact absolutely simple real Lie group and  $K$  is a maximal compact subgroup.
- (ii) The duality defined in 7.83 interchanges spaces of type I and III in the same order.

*ie.  $\mathfrak{g} \oplus \mathbb{R} = \mathfrak{g} \oplus \mathbb{R}$  is simple*

7.105 Table 4. (Non-compact) irreducible symmetric spaces of type IV

$\mathfrak{g}$	$\mathfrak{k}$	$\dim M$	Condition	$\mathfrak{g}$	$\mathfrak{k}$	$\dim M$
$\mathfrak{sl}(n, \mathbb{C})$	$\mathfrak{su}(n)$	$n^2 - 1$	$2 \leq n$	$F_4^{\mathbb{C}}$	$F_4$	52
$\mathfrak{so}(n, \mathbb{C})$	$\mathfrak{so}(n)$	$\frac{n(n-1)}{2}$	$7 \leq n$	$E_6^{\mathbb{C}}$	$E_6$	78
$\mathfrak{sp}(n, \mathbb{C})$	$\mathfrak{sp}(n)$	$n(2n+1)$	$2 \leq n$	$E_7^{\mathbb{C}}$	$E_7$	133
$G_2^{\mathbb{C}}$	$G_2$	14		$E_8^{\mathbb{C}}$	$E_8$	248

Notes

- (i) Here  $M = G/K$  where  $G$  is a simple complex Lie group and  $K$  a maximal compact subgroup.
- (ii) The duality defined in 7.83 interchanges spaces of type II and IV in the same order.