

7.102 Table 1. (Compact) irreducible symmetric spaces of type I

\mathfrak{g}	\mathfrak{k}	$\dim M$	Condition	\mathfrak{g}	\mathfrak{k}	$\dim M$
$\mathfrak{su}(p+q)$	$\mathfrak{su}(p) \oplus \mathfrak{su}(q) \oplus \mathbb{R}$	$2pq$	$1 \leq p \leq q$	E_6	$\mathfrak{su}(6) \oplus \mathfrak{su}(2)$	40
$\mathfrak{su}(n)$	$\mathfrak{so}(n)$	$\frac{(n-1)(n+2)}{2}$	$3 \leq n$	E_6	$\mathfrak{so}(10) \oplus \mathbb{R}$	32
$\mathfrak{su}(2n)$	$\mathfrak{sp}(n)$	$(n-1)(2n+1)$	$2 \leq n$	E_6	$\mathfrak{sp}(4)$	42
$\mathfrak{so}(2n)$	$\mathfrak{su}(n) \oplus \mathbb{R}$	$n(n-1)$	$5 \leq n$	E_6	F_4	26
$\mathfrak{so}(p+q)$	$\mathfrak{so}(p) \oplus \mathfrak{so}(q)$	pq	$1 \leq p \leq q$ $7 \leq p+q$	E_7	$\mathfrak{su}(8)$	70
$\mathfrak{sp}(n)$	$\mathfrak{su}(n) \oplus \mathbb{R}$	$n(n+1)$	$2 \leq n$	E_7	$\mathfrak{so}(12) \oplus \mathfrak{su}(2)$	64
$\mathfrak{sp}(p+q)$	$\mathfrak{sp}(p) \oplus \mathfrak{sp}(q)$	$4pq$	$1 \leq p \leq q$	E_7	$E_6 \oplus \mathbb{R}$	54
G_2	$\mathfrak{su}(2) \oplus \mathfrak{su}(2)$	8		E_8	$\mathfrak{so}(16)$	128
F_4	$\mathfrak{so}(9)$	16		E_8	$E_7 \oplus \mathfrak{su}(2)$	112
F_4	$\mathfrak{sp}(3) \oplus \mathfrak{su}(2)$	28				

7.103 Table 2. (Compact) irreducible symmetric spaces of type II

\mathfrak{k}	$\dim M$	Condition	\mathfrak{k}	$\dim M$
$\mathfrak{su}(n)$	$n^2 - 1$	$2 \leq n$	F_4	52
$\mathfrak{so}(n)$	$\frac{n(n-1)}{2}$	$7 \leq n$	E_6	78
$\mathfrak{sp}(n)$	$n(2n+1)$	$2 \leq n$	E_7	133
G_2	14		E_8	248

Notes

- (i) Here $\mathfrak{g} = \mathfrak{k} \oplus \mathfrak{k}$, and the embedding $\mathfrak{k} \subset \mathfrak{g}$ is the diagonal one.
- (ii) M is a (connected) simple compact Lie group.