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Institut für Algebra und Geometrie
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Algebra II – Problem Sheet 5

Exercise 1 (4 points)

a) Let L/K be an algebraic and separable field extension. Show that $\Omega_{L/K} = 0$.

b) Let K be a field of characteristic $p > 0$ and c be an element of K which is no p-power in K. Let $L = K[X]/(X^p - c)$ be the splitting field of $f := X^p - c$. Show that $\Omega_{L/K} \cong L$.

Exercise 2 (4 points)
Let K be a field of characteristic 0 and $A := K[X_1, \ldots, X_n]$. Show that $H^n_{\mathrm{dR}}(A) = 0$.

Exercise 3 (4 points)
Determine $\Omega_{A/R}$ for $A := R[X,Y]/(XY)$.

Exercise 4 (4 points)
Let A be an R-algebra and define an A-homomorphism $\Phi : A \otimes_R A \to A$ by $\Phi(a \otimes b) := ab$. Let I be the kernel of Φ. Show that the map

$$\varphi : \Omega_{A/R} \to I/I^2, \quad da \mapsto (1 \otimes a - a \otimes 1) + I^2$$

is an isomorphism of A-modules.

Hint: Consider the ring $B := \{(a, x) \,:\, a \in A,\ x \in \Omega_{A/R}\}$ in which the operations are given by $(a_1, x_1) + (a_2, x_2) := (a_1 + a_2, x_1 + x_2)$ and $(a_1, x_1) \cdot (a_2, x_2) := (a_1 a_2, a_1 x_2 + a_2 x_1)$ $(a_1, a_2 \in A,\ x_1, x_2 \in \Omega_{A/R})$. Use the R-algebra-homomorphism $\psi : A \otimes_R A \to B$, $a \otimes b \mapsto (ab, a\,db)$.

Solutions to be handed in on Tuesday, 20. 5. 2008, at the beginning of the problem session in S12.