UNIVERSITÄT KARLSRUHE
INSTITUT FÜR ALGEBRA UND GEOMETRIE
PROF. DR. F. HERRLICH
UTE WOLF

20. 5. 2008

Algebra II – Problem Sheet 6

Exercise 1 (4 points)
Let $J, I_1, \ldots, I_n \trianglelefteq R$ be ideals such that at most two of the I_j are not prime. Show that if $J \subseteq I_1 \cup \ldots \cup I_n$ then J is contained in one of the I_j.

Hint: Use induction on n and do the case $n = 2$ separately.

Exercise 2 (4 points)
Show that $R[[X]]$ is noetherian if R is noetherian.

Exercise 3 (4 points)
Let R be a ring such that for any maximal ideal m, the localized ring R_m is noetherian. Furthermore, suppose that any element $r \in R \setminus \{0\}$ is contained in only finitely many maximal ideals. Show that R is noetherian.

Hint: To show that an ideal I is finitely generated, choose an arbitrary element $x \in I \setminus \{0\}$ and consider the localizations of R at all maximal ideals containing x.

Exercise 4 (4 points)
An R-module M is called *artinian* if and only if any decreasing sequence of submodules of M eventually becomes stationary. The ring R is called *artinian* if it is artinian as R-module. Show that:

a) If

$$0 \to M' \to M \to M'' \to 0$$

is a short exact sequence of R-modules, then M is artinian if and only if M' and M'' are artinian.

b) Every integral artinian ring is a field.

c) In an artinian ring, every prime ideal is maximal.

Solutions to be handed in on Tuesday, 27. 5. 2008, at the beginning of the problem session in S12.