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Algebra II – Problem Sheet 7

Exercise 1 (4 points)
Let $d \in \mathbb{Z} \setminus \{0, 1\}$ be a square-free integer. Show that the integral closure $\mathcal{O}_d$ of $\mathbb{Z}$ in $\mathbb{Q}(\sqrt{d})$ is the following:

$$\mathcal{O}_d = \begin{cases} \mathbb{Z}[\sqrt{d}] & , d \equiv 2, 3 \mod 4 \\ \mathbb{Z}[\frac{1+\sqrt{d}}{2}] & , d \equiv 1 \mod 4 \end{cases}$$

Exercise 2 (4 points)
Let $R \subseteq S$ be integral domains such that S/R is an integral ring extension. Show that for any nonzero ideal $(0) \neq I \trianglelefteq S$ of S, the set $I \cap R$ is a nonzero ideal of R.

Exercise 3 (4 points)
Let R be a ring and $G \leq \mathrm{Aut}(R)$ be a finite subgroup of the automorphism group of R. Show that

$$R^G := \{r \in R \mid \forall\, g \in G : \; g(r) = r\}$$

is a subring of R and that R/R^G is an integral ring extension.

Exercise 4 (4 points)
Let R be an integral domain and $S \subseteq R$ be a multiplicatively closed subset of R such that $1 \in S$ and $0 \notin S$. Show that R_S is normal if R is normal.

Solutions to be handed in on Tuesday, 3. 6. 2008, at the beginning of the problem session in S12.