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Algebra II – Problem Sheet 11

On this problem sheet, let K be a field.

Exercise 1 (4 points)
Let R be a noetherian, local ring and $I \trianglelefteq R$, $I \neq R$ be a proper ideal of R. Show that R is integral if $\mathrm{gr}_I(R)$ is integral. (For the definition of $\mathrm{gr}_I(R)$, compare problem sheet 8.)

Exercise 2 (4 points)
Determine a Noether normalization of the K-algebra

$$A := K[X, Y, Z]/(XY + Z^2, X^2Y - XY^3 + Z^4 - 1)\,.$$

Exercise 3 (4 points)
Let A be a finitely generated K-algebra with generators $a_1, \ldots, a_n$. Let $F \in K[X_1, \ldots, X_n] \setminus \{0\}$ be a polynomial such that $F(a_1, \ldots, a_n) = 0$. Show that there are $\mu_1, \ldots, \mu_{n-1} \in \mathbb{N}$ such that $F(X_1 + X_n^{\mu_1}, \ldots, X_{n-1} + X_n^{\mu_{n-1}}, X_n)$ has constant leading coefficient (as polynomial in X_n).

Exercise 4 (4 points)
Let A be a finitely generated, integral K-algebra and $P \trianglelefteq A$ be a prime ideal of A with $\mathrm{ht}(P) = 2$. Show that there are infinitely many prime ideals $Q \subseteq P$ in A of height 1.

Solutions to be handed in on Tuesday, 1. 7. 2008, at the beginning of the problem session in S12.