

Algebra – Problem Sheet 2

Exercise 1 (4 points)

Let G be any group. Show that the following statements are equivalent:

1. G is solvable.
2. There is a normal subgroup N of G with N and G/N solvable.
3. All subgroups and factor groups of G are solvable.

Exercise 2 (5 points)

Show that all groups of order < 60 are solvable. Use statements from the lecture and exercise 1.

Hint: Sylow might help you.

Exercise 3 (4 points)

Let G be any group. Remember that the commutator subgroup $[G, G]$ of G is defined as

$$[G, G] := \langle ghg^{-1}h^{-1} : g, h \in G \rangle.$$

Now we inductively define $G^{(0)} := G$ and $G^{(i+1)} = [G^{(i)}, G^{(i)}]$ for all $i \in \mathbb{N}_0$.

Show that G is solvable if and only if there is an i with $G^{(i)} = \{e\}$.

Exercise 4 (3 points)

Let G be any group and $H, N \trianglelefteq G$, $H \neq N$ be normal subgroups, such that $1 \trianglelefteq H \trianglelefteq G$ and $1 \trianglelefteq N \trianglelefteq G$ are composition series. Show that $G \cong H \times N$.

Hint: Look at the proof of the Jordan Hölder theorem.

Hand in your solutions until wednesday, November 2nd 2011, 7:45 in the yellow box labeled „Algebra“, Allianzgebäude, 1C or bring them directly to the problem class, 8:00.