Karlsruhe Institute of Technology
Institute for Algebra und Geometry
Dr. R. Dahmen
M. Sc. L. Weber

46	47	48	49	50	$\sum$

Karlsruhe, December 20, 2019

Name: .....

Exercise Sheet No. 10
Advanced Mathematics I

Exercise 46:

Determine the derivatives of

(i) $f_1(x) = x^x, \quad x \in \mathbb{R}_{>0},$

(ii) $f_2(x) = (\sqrt{x}+1)\left(\frac{1}{\sqrt{x}} - 1\right), \quad x \in \mathbb{R}_{>0},$

(iii) $f_3(x) = \frac{\sin x}{\sin x + \cos x}, \quad x \in \left[0, \frac{\pi}{2}\right]$

(iv) $f_4(x) = e^{(\sin x)^2} + e^{\sin(x^2)} + \left(e^{\sin x}\right)^2, \quad x \in \mathbb{R}.$

Exercise 47:

Consider the function

$$f(x) = \begin{cases} e^x, & \text{if } x \leq 0, \\ \cos(x) + x, & \text{if } x > 0. \end{cases}$$

(a) Show that f is continuously differentiable at $x \neq 0$ arbitrarily many times.

(b) Show that f is (once) continuously differentiable at $x = 0$.

(c) Is f twice continuously differentiable at $x = 0$?

Exercise 48:

(a) Show that the function given by $f(x) = \sinh(x)$ is strongly monotonically increasing on $\mathbb{R}$ and is thus injective.

(b) Show that f is bijective, and thus has an inverse.

(c) Use the representation $\sinh(x) = \frac{1}{2}(e^x - e^{-x})$ to determine the inverse $f^{-1}(y) = \operatorname{Arsinh}(y)$ and its derivative $(f^{-1})'(y)$.

Exercise 49:

Prove the following inequalities:

(a) $\ln x > \frac{2(x-1)}{x+1}$ for all $x \in \mathbb{R}$, $x > 1$,

(b) $5x + \frac{1}{x^5} \geq 6$ for all $x \in \mathbb{R}$, $x > 0$.

Exercise 50:

Consider the function $f : [-5, 5] \to \mathbb{R}$ and $g : [\frac{1}{2}, \frac{5}{2}] \to \mathbb{R}$ given by

$$f(x) = \frac{x+2}{x^2+4x+5} \qquad \text{and} \qquad g(x) = 2x^3 - 9x^2 + 12x.$$

Determine the images of f and g.

Due date: Your written solutions are due at 12:00 on Monday, **January 20, 2020**. Please submit them in the green box labelled "AM1" in the atrium of the maths building (20.30).

Website: For detailed information regarding this course visit the following web page:

`http://www.math.kit.edu/iag3/edu/am12019w/en`