Karlsruhe Institute of Technology
Institute for Algebra und Geometry
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56	57	58	59	60	$\sum$

Karlsruhe, January 24, 2020

Name: .....

Exercise Sheet No. 12
Advanced Mathematics I

Exercise 56:

Use integration by parts in order to calculate the following indefinite integrals

$$\text{(a)} \quad \int \sin^2(x)\,dx, \qquad \text{(b)} \quad \int x^2 \cdot \ln(x)\,dx.$$

Use integration by parts in order to calculate the following definite integrals

$$\text{(c)} \quad \int_0^1 x \cdot \arctan(x)\,dx, \qquad \text{(d)} \quad \int_0^{\frac{\pi}{2}} \cos^4(x)\,dx.$$

Exercise 57:

Use integration by substitution to calculate the following anti-derivatives

$$\text{(a)} \quad \int \frac{1}{x \ln x}\,dx \text{ on } (1,\infty), \qquad \text{(b)} \quad \int \frac{x}{\sqrt{x^2-1}}\,dx \text{ on } (1,\infty).$$

Calculate the following integral using first substitution and then integration by parts

$$\text{(c)} \quad \int_1^4 \arctan\sqrt{\sqrt{\mathrm{x}}-1}\,\mathrm{dx}.$$

Exercise 58:

Find the derivative F' of the following function:

$$F : [0,1] \to \mathbb{R}, \quad F(x) = \int_{\ln(x)}^{x^2} \sin(\cos(t))\,dt.$$

Exercise 59:

Solve the following initial value problem

$$y'(x) - y(x) - 2xe^x = 0, \quad y(0) = 1.$$

Exercise 60:

Solve the initial value problem

$$y^3(x) - x^2 + xy^2(x)y'(x) = 0, \qquad y(1) = 1.$$

Hint: Link this problem to Bernoulli-type ODEs.

Due date: Your written solutions are due at 12:00 on Monday, **February 3, 2020**. Please submit them in the green box labelled "AM1" in the atrium of the maths building (20.30).

Website: For detailed information regarding this course visit the following web page:

`http://www.math.kit.edu/iag3/edu/am12019w/en`