Karlsruhe Institute of Technology
Institute for Algebra und Geometry
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Karlsruhe, October 25, 2019

Name: ........................

# Exercise Sheet No. 2
# Advanced Mathematics I

**Exercise 6:**

Solve the following inequalities for $x$ ($x \in \mathbb{R}$):

(a) $(x-5)^3(x+1) \geq 0,$ (b) $\dfrac{(x+1)(3-x)}{(x+5)^2} \leq 0,$ (c) $|x| = x^3 + 2x^2 - 3x.$

**Exercise 7:**

Prove the following identities:

(a) $\binom{n}{r} = \frac{n}{r} \cdot \binom{n-1}{r-1}$ , $n \geq r \geq 1$ ;

(b) $\binom{n}{m} \cdot \binom{m}{r} = \binom{n}{r} \cdot \binom{n-r}{m-r}$ , $n \geq m \geq r \geq 0$ .

**Exercise 8:** Prove by induction on $n \in \mathbb{N}$ that

(a) $\displaystyle\sum_{k=1}^{n} \frac{1}{k^2} \leq 2 - \frac{1}{n},$ (b) $\displaystyle\sum_{k=1}^{n} \frac{1}{(3k-2)(3k+1)} = \frac{n}{3n+1}.$

**Exercise 9:** Consider the following identity

$$\sum_{k=1}^{n} \frac{(2k)! - 2\big((2k-2)!\big)}{2^k} = \frac{(2n)!}{2^n} - 1.$$

(a) Prove this identity for $n \in \mathbb{N}$ by induction.

(b) Prove this identity for $n \in \mathbb{N}$ by shifting the index.

**Exercise 10:** Prove by induction on $n \in \mathbb{N}$ that

(a) $2^n > 2n+1,$ for $n \geq 3$ (b) $2^n \geq n^2,$ for $n \geq 4$ (c) $\displaystyle\sum_{l=0}^{n} \binom{n}{l} = 2^n.$

**Due date:** Your written solutions are due at 12:00 on Monday, **November 4, 2019**. Please submit them in the green box labelled “AM1” in the atrium of the maths building (20.30).
**Problem Session:** 15:45 Friday, October 25, 2019
**Website:** For detailed information regarding this course visit the following web page:

`http://www.math.kit.edu/iag3/edu/am12019w/en`