Exercise 6:
Solve the following inequalities for $x$ ($x \in \mathbb{R}$):

(a) $(x - 5)^3(x + 1) \geq 0$,  
(b) $\frac{(x + 1)(3 - x)}{(x + 5)^2} \leq 0$,  
(c) $|x| = x^3 + 2x^2 - 3x$.

Exercise 7:
Prove the following identities:

(a) \[ \binom{n}{r} = \frac{n}{r} \binom{n - 1}{r - 1}, \quad n \geq r \geq 1; \]

(b) \[ \binom{n}{m} \cdot \binom{m}{r} = \binom{n}{r} \cdot \binom{n - r}{m - r}, \quad n \geq m \geq r \geq 0. \]

Exercise 8: Prove by induction on $n \in \mathbb{N}$ that

(a) \[ \sum_{k=1}^{n} \frac{1}{k^2} \leq 2 - \frac{1}{n}, \]

(b) \[ \sum_{k=1}^{n} \frac{1}{(3k - 2)(3k + 1)} = \frac{n}{3n + 1}. \]

Exercise 9: Consider the following identity

\[ \sum_{k=1}^{n} \frac{(2k)! - 2((2k - 2)!)}{2^k} = \frac{(2n)!}{2^n} - 1. \]

(a) Prove this identity for $n \in \mathbb{N}$ by induction.

(b) Prove this identity for $n \in \mathbb{N}$ by shifting the index.

Exercise 10: Prove by induction on $n \in \mathbb{N}$ that

(a) $2^n > 2n + 1$, for $n \geq 3$  
(b) $2^n \geq n^2$, for $n \geq 4$  
(c) $\sum_{l=0}^{n} \binom{n}{l} = 2^n$.

Due date: Your written solutions are due at 12:00 on Monday, November 4, 2019. Please submit them in the green box labelled “AM1” in the atrium of the maths building (20.30).

Problem Session: 15:45 Friday, October 25, 2019
Website: For detailed information regarding this course visit the following web page:

http://www.math.kit.edu/iag3/edu/am12019w/en