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Exercise Sheet No. 3 Advanced Mathematics I

Exercise 11: Given the complex numbers

$$(i) z = 3 - i, \quad (ii) z = 3 + 4i,$$

resolve each of the numbers \bar{z} , $-z$, $z\bar{z}$, $\frac{1}{z}$, $z - \bar{z}$, $|z|$ into their real and imaginary parts and plot them in the complex plane.

Exercise 12:

(a) Compute a polar coordinate representation $r(\cos(\varphi) + i\sin(\varphi))$ for each of the following numbers:

$$(i) i, \quad (ii) -1 + i, \quad (iii) \frac{1 - i}{i + 2}.$$

(b) Let $z, w \in \mathbb{C}$. Compute $\operatorname{Re}(z\bar{w})$ by

- (i) using Cartesian coordinates: $z = x + iy$, $w = a + ib$ with $x, y, a, b \in \mathbb{R}$.
- (ii) using polar coordinates: $z = r(\cos(\varphi) + i\sin(\varphi))$, $w = q(\cos(\psi) + i\sin(\psi))$ with $r, q \geq 0$, $\varphi, \psi \in (-\pi, \pi]$.

Exercise 13: Sketch the following subsets of the complex plane:

- (a) $A = \{z \in \mathbb{C} : |\operatorname{Re} z| \leq 3, \operatorname{Im} z \leq 2\}$,
- (b) $B = \{z \in \mathbb{C} : 1 \leq |z| \leq 2, \operatorname{Arg} z \in (-\pi, -\frac{\pi}{2}] \cup [\frac{3\pi}{4}, \pi]\}$,
- (c) $C = \{z \in \mathbb{C} : |z|^2 \leq 2\operatorname{Im} z\}$.

Exercise 14: Find all solutions $z \in \mathbb{C}$ of the equation by completing the square:

$$z^2 + (2 - i2\sqrt{2})z - 7 - i(8 + 2\sqrt{2}) = 0.$$

Exercise 15: Determine all complex solutions z of the following equations:

- (a) $z^2 + (2i + 2)z + 4i = 0$,
- (b) $z^4 + (2i + 2)z^2 + 4i = 0$.

Due date: Your written solutions are due at 12:00 on Monday, **November 11, 2019**. Please submit them in the green box labelled "AM1" in the atrium of the maths building (20.30).

Problem Session: 8:00 Wednesday, November 06, 2019

Website: For detailed information regarding this course visit the following web page:

<http://www.math.kit.edu/iag3/edu/am12019w/en>