Karlsruhe Institute of Technology
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Karlsruhe, November 8, 2019

Name: .....

# Exercise Sheet No. 4
# Advanced Mathematics I

**Exercise 16:**

(a) Give a representation of $\mathrm{i}+1$ in polar coordinates. Use this representation to find all solutions $z \in \mathbb{C}$ of the equality

$$z^2 = \mathrm{i}+1.$$

(b) Find all solutions $z \in \mathbb{C}$ of the following equation by means of completing the square

$$z^2 + (3+5\mathrm{i})z - \left(3 - \tfrac{17}{2}\mathrm{i}\right) = 0.$$

**Exercise 17:** Consider the sequence $(a_n)_n$ with $a_n = \frac{n-1}{n+1}$, $n \in \mathbb{N}$. Find an index $N$ such that $|a_n - 1| \leq \varepsilon$ for every $n \geq N$, when

(a) $\varepsilon = \dfrac{1}{10}$, (b) $\varepsilon = \dfrac{1}{1000}$, (c) $\varepsilon > 0$ is arbitrary.

(d) Does the sequence $(a_n)_n$ converge? If so, what is the limit?

**Exercise 18:** Consider the sequence

$$a_n = \frac{1}{2} + (-1)^n \left(1 - \frac{1}{n}\right).$$

(a) Is the sequence bounded? If so, give a value for $r$ for which $|a_n| \leq r$.

(b) Give the smallest possible such $r$; justify your answer.

**Exercise 19:** Test the following sequences for convergence:

$$a_n = \frac{n}{n^2-2}, \qquad b_n = \frac{n^2-2}{n}, \qquad c_n = n-1,$$

$$d_n = b_n - c_n, \qquad e_n = \frac{2+(-1)^{n-1}}{n+7}.$$

**Exercise 20:** Determine the limit of the sequence $(a_n)_n$, where

(a) $a_n = \sqrt[n]{4 + \dfrac{n-1}{n+1}}$,

(b) $a_n = \dfrac{n^4-2}{n^2+4} + \dfrac{n^3(3-n^2)}{n^3+1}$,

(c) $a_n = \sqrt[n]{34^n + 118^n} \cdot \left[\dfrac{(n+4)^4}{n^3} - n + 1\right] + 3$.

**Due date:** Your written solutions are due at 12:00 on Monday, **November 18, 2019**. Please submit them in the green box labelled "AM1" in the atrium of the maths building (20.30).

**Problem Session:** 8:00 Wednesday, November 13, 2019

**Website:** For detailed information regarding this course visit the following web page:

`http://www.math.kit.edu/iag3/edu/am12019w/en`