Karlsruhe Institute of Technology
Institute for Algebra und Geometry
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Karlsruhe, November 15, 2019

Name: ....

Exercise Sheet No. 5
Advanced Mathematics I

Exercise 21: Which of the following assertions are true? Justify correct ones and give a counter example for each incorrect assertion.

(a) If a sequence is monotone and bounded, then it converges.

(b) If a sequence converges, then it is monotone and bounded.

(c) If a sequence is not bounded, then it is not convergent.

(d) If a sequence is not monotone, then it is not convergent.

(e) If a sequence has exactly one accumulation point, then it converges.

(f) If a sequence converges, then it has exactly one accumulation point.

Exercise 22:

Let the sequence (a_n) be given by a starting value $a_0 \in [0,2]$ and the recursion

$$a_{n+1} = \frac{a_n\,(a_n^2+3)}{3a_n^2+1}, \qquad n = 0, 1, 2, \ldots.$$

(a) Show that

$$a_{n+1} - 1 = \frac{(a_n-1)^3}{3a_n^2+1}, \qquad n = 0, 1, 2, \ldots.$$

Also prove the following two statements:

$$0 < a_0 < 1 \implies 0 < a_n < 1 \text{ for all } n \in \mathbb{N},$$
$$1 < a_0 < 2 \implies 1 < a_n < 2 \text{ for all } n \in \mathbb{N}.$$

(b) Show that the sequence is strictly montonically increasing for $0 < a_0 < 1$ and strictly monotonically decreasing for $1 < a_0 < 2$.

(c) For which $a_0 \in [0,2]$ does the sequence converge? If so, determine the limit.

Exercise 23:

Split the sequence $(a_n)_n$, given by

$$a_n = \frac{1+2^n}{1+2^n+(-2)^n}, \qquad n \in \mathbb{N},$$

into appropriate subsequences and test those for monotonicity, boundedness and convergence. Does the sequence (a_n) converge?

Exercise 24: Let f be a real-valued function defined by

$$f(x) = \frac{x^3-3x+2}{x^3-7x+6}.$$

Determine the maximal domain D and the range $f(D)$ of f.

Exercise 25:

Consider the following polynomial: $f(x) = x^4 + 5x^3 - 8x^2 + 1 - (x-2)^3$.

(a) Expand f about the expansion points $x_1 = 1$ and $x_2 = -1$.

(b) Determine all zeros x_i of f by factoring out factors of the form $(x - x_i)$.

Due date: Your written solutions are due at 12:00 on Monday, **November 25, 2019**. Please submit them in the green box labelled “AM1” in the atrium of the maths building (20.30).
Problem Session: 8:00 Wednesday, November 20, 2019
Website: For detailed information regarding this course visit the following web page:

`http://www.math.kit.edu/iag3/edu/am12019w/en`