Karlsruhe Institute of Technology
Institute for Algebra und Geometry
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26	27	28	29	30	$\sum$

Karlsruhe, November 22, 2019

Name: ....

Exercise Sheet No. 6
Advanced Mathematics I

Exercise 26:

Consider the polynomial $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) := \frac{1}{8}\,x^3 + \frac{3}{8}\,x^2 - \frac{9}{8}\,x + \frac{5}{8}$.

(a) Expand f about the expansion points $x_1 = 1$ and $x_2 = -3$. Use this representation to discuss the behavior of f on the interval $[1, \infty)$.

(b) Use a sketch of f to find intervals on which f has an inverse function. Also sketch the inverse.

Exercise 27:

Let $c \in \mathbb{R}$ and consider the function $f : \mathbb{R} \to \mathbb{R}$ given by

$$f(x) = \begin{cases} 1 - 2x - x^2, & x < 0, \\ c(x-2)^2, & x \geq 0. \end{cases}$$

(a) Find a value of c such that f is continuous and give a sketch of $f(x)$ for this c on the interval $[-3, 4]$.
For the remaining exercise study f for this fixed c.

(b) Find all maximal intervals I of $\mathbb{R}$ where f is invertible.
Note: Here maximal means that there is no interval I' with $I \subsetneq I'$ and f is invertible on I'.
For each of these intervals give the inverse of f and the domain of the inverse on this interval.

(c) Find a maximal domain $D \subseteq \mathbb{R}$ such that the function $f : D \to \mathbb{R}$ is bijective.

Exercise 28:

Show that the function

$$x \mapsto \frac{1}{\sqrt{x}}$$

is continuous on the domain $D = (0, \infty)$. Sketch the graph of the function.

Exercise 29:

For each of the following functions $f_j : \mathbb{R} \to \mathbb{R}$ find all points $x \in \mathbb{R}$ where f_j is continuous

(a) $$f_1(x) := \begin{cases} \dfrac{x^3 + 4x^2 + x - 6}{x^3 - 3x + 2}, & x \in \mathbb{R} \setminus \{1, -2\}, \\ 0, & x = 1, \\ -\frac{1}{3}, & x = -2, \end{cases}$$

(b) $$f_2(x) := \begin{cases} x, & x \in \mathbb{Z}, \\ 0, & \text{otherwise.} \end{cases}$$

Exercise 30:

Prove that the function

$$x \mapsto \sqrt[3]{x}$$

is continuous on the domain $D = [0, \infty)$.
Hint: For the case $x \neq 0$ use the identity $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$ for $a, b \in \mathbb{R}$.

Due date: Your written solutions are due at 12:00 on Monday, **December 2, 2019**. Please submit them in the green box labelled "AM1" in the atrium of the maths building (20.30).
Problem Session: 8:00 Wednesday, November 27, 2019
Website: For detailed information regarding this course visit the following web page:

`http://www.math.kit.edu/iag3/edu/am12019w/en`