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Exercise Sheet No. 7 Advanced Mathematics I

Exercise 31:

Decide for each of the following subsets of \mathbb{R} if they are bound, open, closed and compact:

- (a) $A = (-13, 3]$, (b) $B = ([-4, 7] \cup [10, \infty)) \cap (-12, 15]$, (c) $C = [-\frac{7}{2}, \infty) \setminus \mathbb{N}$.

Exercise 32: Consider the function $f : [-2, 2] \rightarrow \mathbb{R}$ given by

$$f(x) = 1 - 2x - x^2.$$

Show that there exist a maximum x_+ and a minimum x_- . Determine x_+ and x_- .

Exercise 33:

- (a) By means of Bolzano's intermediate value theorem show that the set $M = \{\sqrt{(1-x^2)} : x \in [-1, 1/2]\} \subset \mathbb{R}$ is compact.
- (b) How many solutions does the equation

$$2x^5 - 6x^3 + 2x = 4x^4 - 6x^2 + 1$$

have in the interval $I = [-2, 2]$? Justify your answer!

Hint: Find an appropriate function f s.t. the equality can be written as $f(x) = 0$ and evaluate f at the points $-2, -1, \frac{1}{2}, 1, 2, 3$.

Exercise 34:

Consider the functions $f, g : \mathbb{R} \rightarrow \mathbb{R}$ given by

$$f(x) = \begin{cases} -x^2 - 8x - 4, & x \leq -3, \\ x^2 - 3, & x > -3, \end{cases} \quad \text{and} \quad g(x) = -x - 2, \quad x \in \mathbb{R}.$$

Show that the graphs of the two functions have at least 3 intersection points.

Hint: You do not need to determine the intersection points.

Exercise 35:

Consider the following series

(a) $\left(\sum_{k=0}^{\infty} \left(\frac{2}{3}\right)^k \right)$, (b) $\left(\sum_{k=2}^{\infty} \frac{4k}{(k^2-1)^2} \right)$.

Determine in each case the first three partial sums and find a general representation for the n th partial sum s_n . Do the series converge? If so, what is the sum?

Hint for (b): You may use the identity $\frac{4k}{(k+1)^2(k-1)^2} = \frac{(k+1)^2 - (k-1)^2}{(k+1)^2(k-1)^2}$, $k \in \mathbb{N}$.

Due date: Your written solutions are due at 12:00 on Monday, **December 9, 2019**. Please submit them in the green box labelled "AM1" in the atrium of the maths building (20.30).

Problem Session: 8:00 Wednesday, December 4, 2019

Website: For detailed information regarding this course visit the following web page: