Karlsruhe Institute of Technology
Institute for Algebra und Geometry
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36	37	38	39	40	$\sum$

Karlsruhe, December 6, 2019

Name: ....

Exercise Sheet No. 8
Advanced Mathematics I

Exercise 36:

Test the following series for convergence. Determine the value of the series in part (a).

$$\text{(a)}\quad \left(\sum_{k=0}^{\infty}\left(\frac{3+4i}{6}\right)^k\right), \quad \text{(b)}\quad \left(\sum_{k=1}^{\infty}\frac{1}{\sqrt[k]{k+1}}\right), \quad \text{(c)}\quad \left(\sum_{k=1}^{\infty}\frac{1}{k(k+1)}\right).$$

Exercise 37:

Test the following series for convergence using the ratio test:

$$\text{(a)}\quad \left(\sum_{k=0}^{\infty}\frac{2^k\cdot 3^k}{k!\cdot(2k+1)}\right), \qquad \text{(b)}\quad \left(\sum_{k=1}^{\infty}\frac{k!}{2^k+1}\right), \qquad \text{(c)}\quad \left(\sum_{k=0}^{\infty}\frac{k^k}{k!\cdot t^k}\right) \quad \text{for fixed } t\in\mathbb{N}.$$

Exercise 38:

Test the following series for convergence using the root test:

$$\text{(a)}\quad \left(\sum_{k=1}^{\infty}\left(\frac{3}{4}+\frac{1}{k}\mathrm{i}\right)^k\right), \qquad \text{(b)}\quad \left(\sum_{k=1}^{\infty}\left(1+\frac{1}{k}\right)^{k^2}\frac{1}{2^k}\right), \qquad \text{(c)}\quad \left(\sum_{k=1}^{\infty}(-1)^k\frac{k^3}{3^k}\right).$$

Exercise 39:

Use the Leibniz test to show that the following series converge. Moreover find an index N such that for all $n \geq N$ the n^{th} partial sum differs from the value of the series by at most $\frac{1}{100}$.

$$\text{(a)}\quad \left(\sum_{k=1}^{\infty}(-1)^{k+1}\frac{k+1}{k^2+3k+2}\right), \qquad \text{(b)}\quad \left(\sum_{k=1}^{\infty}(-1)^{k+1}\left(\sqrt{2k+2}-\sqrt{2k}\right)\right).$$

Exercise 40:

(a) Show that the series

$$\left(\sum_{k=0}^{\infty}\left(\frac{x-1}{x+1}\right)^k\right), \quad \text{for fixed } x\in\mathbb{R}_{>0}$$

converges and determine the value.

(b) For which $q \in \mathbb{R}$ does the series $\sum_{n=0}^{\infty}(n+1)q^n$ converge?

Due date: Your written solutions are due at 12:00 on Monday, **December 16, 2019**. Please submit them in the green box labelled “AM1” in the atrium of the maths building (20.30). If you want them to be graded over Christmas, please give them to your student tutor during the class on Friday, December 13.

Problem Session: 8:00 Wednesday, December 11, 2019

Website: For detailed information regarding this course visit the following web page:

`http://www.math.kit.edu/iag3/edu/am12019w/en`