

Selected Topics in Geometric Group Theory

Problem Sheet 9

The problems on this sheet are meant as *suggestions* for what to do in your holidays if you really want to do some math. In particular, you are *not required* to work through all the referenced material! However, if you have a look at any of these articles, you are invited to report on your findings in the first problem session in 2013.

Problem 1 *A finite set of Dehn twists generates the mapping class group.*

We saw in the lectures that $\text{Mod}(S_g)$ is generated by Dehn twists only. Actually, a *finite* number of Dehn twists suffices for this purpose. You may find a proof of this fact in the article [Lic64] of Lickorish.

Problem 2 *A minimal set of generating Dehn twists*

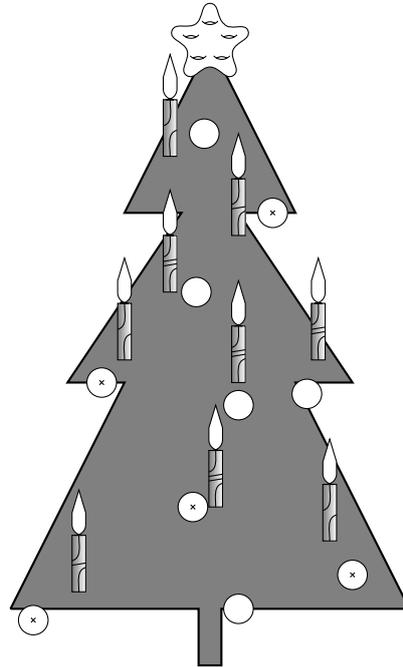
Presupposing the results mentioned in the previous problem, we already saw on the last problem sheet – or at least in the solution to Problem 3 – that $2g + 2$ Dehn twists suffice to generate $\text{Mod}(S_g)$. Actually, this is a minimal generating set of $\text{Mod}(S_g)$ in the sense that no system of Dehn twists with less than $2g + 2$ elements is capable of generating the whole mapping class group. This minimality is proven in the article [Hum79] of Humphries.

Problem 3 *A minimal set of generators*

A rather surprising result, also due to Lickorish, is that only four elements suffice to generate the whole mapping class group $\text{Mod}(S_g)$. Actually, this already tiny number was later reduced to only *two generators* by Wajnryb in his article [Waj96]. As shown by Korkmaz in [Kor05], one of these two generators can indeed be chosen to be a Dehn twist.

Problem 4 *Most importantly, ...*

We wish you a merry Christmas, a happy new year and great holidays!



References

- [Hum79] S. P. Humphries. “Generators for the mapping class group”. In: *Topology of Low-Dimensional Manifolds*. Ed. by Roger Fenn. Vol. 722. LNM. Available online at <http://dx.doi.org/10.1007/BFb0063188>. 1979, pp. 44–47.
- [Kor05] M. Korkmaz. “Generating the surface mapping class group by two elements”. In: *Trans. Amer. Math. Soc.* 357.8 (2005). available online at <http://dx.doi.org/10.1090/S0002-9947-04-03605-0>, 3299–3310 (electronic).
- [Lic64] W. B. R. Lickorish. “A finite set of generators for the homeotopy group of a 2-manifold”. In: *Proceedings of the Cambridge Philosophical Society* 60 (1964). Available online at <http://dx.doi.org/10.1017/S030500410003824X>, p. 769.
- [Waj96] B. Wajnryb. “Mapping class group of a surface is generated by two elements”. In: *Topology* 35.2 (1996). available online at [http://dx.doi.org/10.1016/0040-9383\(95\)00037-2](http://dx.doi.org/10.1016/0040-9383(95)00037-2), pp. 377–383.