Karlsruher Institut für Technologie

Institute for Algebra and Geometry
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Wintersemester 2012/2013: Course on the topic

Selected Topics in Geometric Group Theory: The mapping class group

One of the main ideas of geometric group theory is to study finitely generated groups by their action on geometric spaces via symmetries and to link algebraic properties of the groups to geometric properties of the spaces they are acting on.

A main actor in this field is the famous *mapping class group of surfaces*. This is the group of all homeomorphisms modulo those homeomorphisms „which do almost nothing on the surface“, i.e. are homotopic to the identity. This group can be easily defined, but its structure, algebraic properties and geometry give rise to a rich theory and have lead to interesting developments in the research of the last fifty years. Beyond that it still holds several open questions which could not be solved so far.

The study of mapping class groups connects to very different mathematical disciplines. There are purely combinatorial group theoretical aspects of how to present this finitely presentable group. It gets a strongly geometric and combinatorial flavour, if one studies its actions on combinatorial objects as the *curve complex* or the *pants complex* and its relations to so called *braid groups*. Close relations to the theory of low-dimensional manifolds going back to the fundamental work of Thurston, and the study of cohomological stability questions lead deeply into topology. Finally, an object very related to it is *Teichmüller space* on which it acts as its group of automorphisms. If we take the mapping class group of a closed surface of genus g, the quotient of this action is the moduli space M_g, a classification space of closed Riemann surfaces of genus g, which plays an immanent role in algebraic geometry.

As a charming aspect one finds close similarities and analogies between mapping class groups, linear groups and the outer automorphism groups $\mathrm{Out}(F_n)$ which suggests to study these groups in connection. Topics which arise from this are e.g. the study of *congruence groups* and the so called *property T*.

The themes mentioned above span a wide range, much more than we can do in this class. We will study selected topics from this and start from the scratch by the definition of the mapping class group.

Prerequisites: Basic knowledge in geometry, topology and algebra e.g. from the courses *Grundlagen in Geometrie und Topologie* and *Grundlagen in Algebra und Zahlentheorie*.

classes: Tuesday 11:30-13:00 and Wednesday 8:00-9:30,
exercise sections: Wednesday 14:00-15:30

For more information see also: http://www.math.kit.edu/iag3/edu/compabelvar2012w/en