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Finite Group Schemes – Excercise Sheet 1

Problem 1

Let k be a ring and X a k-functor. We define

$$|X| := \varinjlim_{l} X(l)$$

where l runs over all field extensions of k. In the colimit we identify two points $x_1 \in X(l_1)$ and $x_2 \in X(l_2)$ whenever there is a field l_3 and embeddings $l_1, l_2 \to l_3$ which send x_1 and x_2 to the same element $x_3 \in X(l_3)$ after application of X. We *assume* that $|X|$ is a *set*.

(a) Show that for any subfunctor $Y \subseteq X$ there is a canonical inclusion $|Y| \subseteq |X|$.

(b) Set

$$\mathfrak{O} := \{|U| \mid U \to X \text{ is an open subfunctor}\}.$$

Prove that $(|X|, \mathfrak{O})$ is a topological space, which depends functorially on X.

(c) For any $X = \mathrm{Sp}_k A$, we have a canonical identification $|X| = \mathrm{Spec} A$, the latter denoting the set of prime ideals in A together with the Zarisiki topology.

(d) Let $\mathfrak{O}_X$ denote the sheaf on $|X|$ associated to the presheaf $|U| \mapsto \mathfrak{O}(U)$. If X is a scheme, then $(|X|, \mathfrak{O}_X)$ is a locally ringed space, locally isomorphic to $(\mathrm{Spec}\, \mathfrak{O}(U), \mathfrak{O}_U)$ for $U \to X$ open affine.

Problem 2

Let l/k denote a finite extension of fields.

(a) Prove that the base change functor

$$- \otimes_k l : \quad \underline{\widehat{k - Fun}} \to \underline{\widehat{l - Fun}}$$

on formal functors induces a well defined base change functor

$$- \otimes_k l : \quad \underline{\widehat{k - Sch}} \to \underline{\widehat{l - Sch}}$$

on formal schemes.

(b) Describe the base change functor in terms of profinite k-algebras and also in terms of k-coalgebras.

(c) Show that the base change functor has a canonical extension to arbitrary field extensions l/k.