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# Finite Group Schemes – Excercise Sheet 2

## Problem 1

Let $k$ be a ring and $X$ a $k$-functor. Assume that we have a morphism $\mu : X \times X \to X$ and $\varepsilon : \mathrm{Sp}_k k \to X$ satisfying the following two axioms: $\mu \circ (\mu \times \mathbf{1}_X) = \mu \circ (\mathbf{1}_X \times \mu)$, $\mu \circ (\varepsilon \times \mathbf{1}_X) = \mu \circ (\mathbf{1}_X \times \varepsilon) = \mathbf{1}_X$. For two triples $\mathfrak{X}_i = (X_i, \mu_i, \varepsilon_i)$, $i \in \{1, 2\}$, we define the set of morphisms $\mathfrak{X}_1 \to \mathfrak{X}_2$ as the subset of morphisms $X_1 \to X_2$ of $k$-functors which render the obvious diagrams involving $\mu_i$ and $\varepsilon_i$ commutative. This gives us a category $\underline{k - Mon}$ of *monoids* over $k$.

Show that there are equivalences between the following categories:

(i) The category $\underline{k - Mon}$.

(ii) The category of functors $\underline{k - Alg} \to \underline{Mon}$ where $\underline{Mon}$ denotes the category of monoids.

Furthermore, the following categories are equivalent:

(i) The subcategory of $\underline{k - Mon}$ consisting of affine $k$-schemes.

(ii) The category of representable functors $\underline{k - Alg} \to \underline{Mon}$ where $\underline{Mon}$ denotes the category of monoids.

(iii) The opposite of the category of $k$-algebras which at the same time are $k$-coalgebras (and the algebra and coalgebra structures are compatible).

Last but not least, for $k$ a field, the following categories are equivalent:

(i) The subcategory of $\underline{k - Mon}$ consisting of étale $k$-schemes.

(ii) The category of monoids equipped with an equivariant $\mathrm{Gal}(k^s/k)$-action.

## Problem 2

Define two explicit $k$-schemes, whose completions become isomorphic in $\widehat{\underline{k - Fun}}$.